 Year 12 Mathematics Standard 2

| MS-N3 Critical path analysis | Unit duration |
| --- | --- |
| Networks involve the graphical representation and modelling of situations as an approach to decision-making processes. Knowledge of networks enables development of a logical sequence of tasks or a clear understanding of connections between people or items. Study of networks is important in developing students’ ability to interpret a set of connections or sequence of tasks as a concise diagram in order to solve related problems.  | 2 weeks |

| Subtopic focus | Outcomes |
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| The principal focus of this subtopic is to use critical path analysis in the optimisation of real-life problems. Students develop awareness that critical path analysis is a useful tool in project planning, management and logistics. | A student:* solves problems using networks to model decision-making in practical problems MS2-12-8
* chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times and methods for such use MS2-12-9
* uses mathematical argument and reasoning to evaluate conclusions, communicating a position clearly to others and justifying a response MS2-12-10

Related Life Skills outcomes: MALS6-11, MALS6-12, MALS6-13, MALS6-14 |

| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| Students should have completed the Stage 6 topic MS-N2 Networks concepts. | Staff could adopt formative assessment strategies and informal diagnostic testing to identify and address students’ misconceptions. |

All outcomes referred to in this unit come from [Mathematics Standard 2019](https://syllabus.nesa.nsw.edu.au/mathematics-standard-stage6/) Syllabus
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| Term | Definition |
| --- | --- |
| Critical path  | The critical path is the sequence of network activities which combine to have the longest overall duration so as to determine the shortest possible time needed to complete a project. |
| Earliest starting time (EST) | The earliest starting time is the earliest time that any activity can be started after all prior activities have been completed. |
| Earliest finish time (EFT) | The earliest time an activity can be finished after all the prerequisite activities have also been finished. |
| Float time  | Float time is the amount of time that a task in a project network can be delayed without causing a delay to subsequent tasks. |
| Flow capacity  | The flow capacity of a network can be found using the maximum-flow minimum-cut theorem and depends upon the capacity of each edge in the network. |
| Latest finish time (LFT) | The latest time an activity can be finished after all the prerequisite activities have also been finished, and with the project still running on time. |
| Latest starting time (LST) | The latest starting time is the latest time an activity may be started after all prior activities have been completed and without delaying the project. |
| Maximum flow minimum cut | The maximum-flow minimum-cut theorem states that the flow through a network cannot exceed the value of any cut in the network and that the maximum flow equals the value of the minimum cut, i.e. it identifies the ‘bottle-neck’ in the system. |

| Lesson sequence | Content | Suggested teaching strategies and resources  | Date and initial | Comments, feedback, additional resources used |
| --- | --- | --- | --- | --- |
| Considerations |  | **Technology note**The following technology can be used throughout this unit to assist students in drawing networks:* [Graph creator](https://illuminations.nctm.org/Activity.aspx?id=3550)
* [Team Gantt](https://www.teamgantt.com/?utm_expid=.Il6xIwhZRhS6iptrhhXPFg.0&utm_referrer)
* [Lucid chart](https://www.lucidchart.com/pages/home)
* [Geogebra](https://www.geogebra.org/graphing) can also be used to create a network diagram with weightings as captions.
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| Constructing networks to model duration and interdependencies(1 lesson) | * construct a network to represent the duration and interdependencies of activities that must be completed during a particular project, for example a student schedule, or preparing a meal **AAM**
 | Introducing networks to model duration and interdependencies* The teacher poses a question or scenario which can be explored to develop an understanding of the duration and interdependencies of tasks. Students need to consider which activities are prerequisites of others and the impact certain activities might have on the overall time. Sample question:
* How long does it really take to get ready for school?

Resource: getting-ready-for-school.DOCXWhen examining this activity, students can consider which activities can be outsourced and therefore can run at the same time. For example, if the student is showering, they could outsource the making of their breakfast to another family member.**Constructing** networks to model duration and interdependencies* There are three different methods for developing and labelling a network diagram to model duration and interdependencies:

Activity table for activities A to G with duration and prerequisites. Activity A: duration 1 day, prerequisites start. Activity B: duration 2 days, prerequisite A. Activity C: duration 3 days, prerequisite B. Activity D: duration 1 day, prerequisite B. Activity E: duration 3 days, prerequisite C. Activity F: duration 2 days, prerequisite D. Activity G: duration 1 day, prerequisites E and F.* Method 1: Attach the activities and the times (weights) to vertices. This is referred to as ‘activity on node’.

This diagram has the activities marked on the vertices and the weights indicated above or below the vertex.* Method 2: Attach the activities and the times (weights) to the edges and start and finish at a vertex. This is referred to as ‘activity on arrow’.

This network diagram has the activities and weights on the edges.* Method 3: Attach the activities to a vertex and the times (weights) to the edges.

This network diagram has the activities placed on the vertices and the times on the edges.Note: In the third diagram, one dummy activity, X, was introduced and the incoming edges have no weight. |  |  |
| Critical path analysis(2 lessons) | * given activity charts, prepare network diagrams and use critical path analysis to determine the minimum time for a project to be completed **AAM**
* use forward and backward scanning to determine the earliest starting time (EST) and latest starting time (LST) for each activity in a project (ACMGM105)
* understand why the EST for an activity could be zero, and in what circumstances it would be greater than zero
* calculate float times of non-critical activities (ACMGM108)
* understand what is meant by critical path
* use ESTs and LSTs to locate the critical path(s) for the project (ACMGM106)
 | Prepare network diagrams from activity charts * The teacher sets the context in terms of project management:
* If you were managing any project you'd create a table that describes each of the activities, their duration and their dependencies or prerequisites.
* Try to establish authentic contexts to use with students

Resource: activity-charts-to-network-diagram.DOCXIntroducing critical paths* The teacher introduces the lesson by posing the question: How can we minimise how long it takes to complete a series of tasks, while maximising the time we have available to do them?
* Students investigate the concept of critical paths using the making a meal activity.

Resource: Making a meal, introducing-critical-paths.DOCX* The teacher defines key terms or concepts for critical path analysis

Resource: key-terms-and-concepts.DOCX**Forward and backward scanning*** The teacher models the identification of a critical path using forward and backward scanning.
* Forward scanning consists of the following steps where a network is constructed using :
* Draw the activity chart with an empty circle at each vertex.
* In each circle write the EST of the activity that starts from that vertex. Assign the start vertex an EST of zero. This means that each activity with no prerequisites can be started immediately.
* Proceed from left to right along each path from the start, writing the EST into each circle as it is encountered. When two or more paths join, select the highest total.
* Continue until the finish vertex is reached. The EST of the finish vertex is the critical time of the project.
* Backward scanning consists of the following steps:
* Begin at the finish vertex.
* In each circle write the LST of any activity that ends at that vertex. Assign the finish vertex an LFT equal to the critical time of the project. Meaning that the latest time the project can finish is also the earliest time it can be finished.
* Proceed from right to left, along each path from the finish, writing the LFT into each circle as it is encountered. When two or more paths join going backwards, select the lowest total.
* Continue until the start vertex is reached. The LFT of the start vertex must always be zero.
* Identify the critical path once forward and backwards scanning have been completed by following the path with zero float times.(i.e.$ EST=LST$)
* Staff may like to use this [Critical Path Analysis Geogebra App](https://www.geogebra.org/m/vvmajjdv) to demonstrate forward and backward scanning techniques to find the critical path.

Resource: critical-path-analysis.PPTX**Forward and backward scanning (alternate method)*** An ‘activity on node’ network diagram can be modified to allow for forward and backward scanning by replacing each vertex with an activity box:

Image of a 3 by 3 table. Top row is EST, blank and EFT. Middle row is Float, Activity and duration. Bottom row is LST, blank and LFT.* Consider the previous sample network diagram:This diagram has the activities marked on the vertices and the weights indicated above or below the vertex.
* Add the activity box:This diagram is the previous network diagram with the vertices replaced with a 3 by 3 activity box. The centre square represents the activity and the centre right square represents the duraiton of the activity.
* Forward scanning:
* Start by entering the EST on node A as 1, to indicate that the process A starts at the start of day 1.

Network diagram showing the start of earliest start time and earliest finish time calculations.* Calculate the EFT as $EFT=EST+duration-1$, for example the EFT for process A is 1. This means that process A needs to be completed by the end of day 1.
* The next process, B, will start the next day so enter the EST on node B as 2 (the EFT of node A plus 1).

Network diagram showing the start of earliest start time and earliest finish time calculations.* Calculate the EFT for process B as above.
* Complete the EST and EFT on each node using forward scanning techniques.

Network diagram showing the start of earliest start time and earliest finish time calculations.* Backward scanning:
* Start by entering the LFT on node G to be equal to the EFT. In the case above this is 10.

Network diagram showing the start of latest start time and latest finish time calculations.* Calculate the LFT as $LST=LFT-duration+1$, therefore in the example above the LST on node G is $10-1+1=10$, which means that process G must start on day 10 (LST) and finish at the end of day 10 (LFT).
* The previous process(es), E and F, will have to finish the day before the LST of process G. Therefore the LFT for nodes E and F will be 9 (the LST of node G – 1).

Network diagram showing the start of latest start time and latest finish time calculations.* Complete the LFT and LST for each node using backward scanning techniques.

Network diagram showing the start of latest start time and latest finish time calculations.NESA exemplar question1. The following table gives details of a set of six tasks which have to be completed to finish a project. The immediate predecessors are those tasks which must be completed before a task may be started.

An image of an table which has 6 activities and indicates their duration and the immediate predecessors for each activity. Activity A: duration of 5 days, predeccessors nil. Activity B: duration of 2 days, predeccessor A. Activity C: duration of 1 day, predeccessor A. Activity D: duration of 2 days, predeccessors B and C. Activity E: duration of 4 days, predeccessors B and C. Activity F: duration of 1 day, predeccessors D and E.  * 1. Draw an activity network
	2. Perform a forward scan and backward scan to find the critical path
	3. What is the critical time for the project?
	4. List the critical activities
	5. Identify the float time of activity $D$
	6. A time lag of one day is needed between activities $B$ and $E$ due to issues of supply of resources. Describe the effect it will have on the critical path.

Resource: ms-n3-nesa-examples-solutions.DOCXApplying critical path analysis* Student activity: Students to use forward and backward scanning to analyse organising a family meal and a get together with friends at the movies. Students are asked to consider how long each activity takes, the earliest time dinner will be ready and what time they will make it to a movie after dinner as well as what if scenarios.

Resource: meal-and-a-movie.DOCX* Student activity: Students can use this [website from brighthubpm.com](https://www.brighthubpm.com/software-reviews-tips/52901-use-excel-to-perform-critical-path-calculations/) to learn how to use Microsoft Excel to determine the critical path and plan a project by developing a Gantt chart.
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| Solve flow problems using network diagrams.(1 lesson) | * solve small-scale network flow problems, including the use of the ‘maximum-flow minimum-cut’ theorem, for example determining the maximum volume of oil that can flow through a network of pipes from an oil storage tank (the source) to a terminal (the sink) (ACMGM109) **AAM**
* convert information presented in a table into a network diagram
* determine the flow capacity of a network and whether the flow is sufficient to meet the demand in various contexts
 | Constructing a network diagram representing flow* Teacher introduces the concept of a network representing flow by showing the [shimap.org](https://www.shipmap.org/) video.
* Driving question: Australia has a trade agreement with China. There are many travel routes between the two countries with different capacities of flow. How do we represent this as a network diagram?
* Student activity: Students draw a network diagram to represent their answers and consider the driving question. Teacher can introduce network terminology (source and sink) by using each student's diagram.
* The graphs should have direction and weightings.

Resource: australias-trade-with-asia.DOCX |  |  |
| Maximum-flow minimum-cut(2 lessons) | * solve small-scale network flow problems, including the use of the ‘maximum-flow minimum-cut’ theorem, for example determining the maximum volume of oil that can flow through a network of pipes from an oil storage tank (the source) to a terminal (the sink) (ACMGM109) **AAM**
* determine the flow capacity of a network and whether the flow is sufficient to meet the demand in various contexts
 | Introducing flows and cuts* The teacher introduces key terms and concepts in network flow diagrams which are networks where the weights on the edges are often referred to as capacities and is the maximum rate of flow through that edge. Terms and concepts include:
* Source
* Sink
* Flow rules
* Excess flow
* Saturated edges
* Cut

Resource: key-terms-and-concepts.DOCXThe maximum-flow minimum-cut theorem* The teacher introduces the concepts of maximum flow and minimum cut.
* The maximum flow from source to sink is called the network’s flow capacity.
* The minimum cut is the cut that has the least capacity.
* The maximum-flow minimum-cut theorem states that the maximum flow through a network is equal to the value of the minimum cut.
* If you have a flow of a certain capacity, and a cut of the same capacity then this theorem says that the flow is the maximum flow, and the cut is the minimum cut. This is useful to show that a cut or flow found by trial and error is actually the best possible.
* A minimum cut will only include edges which are saturated. However, there may be saturated edges which are not part of a minimum cut.
* The teacher models the methods of finding the maximum flow and minimum cut.

Resources: maximum-flow-minimum-cut-methods.DOCX, maximum-flow-minimum-cut-methods.PPTX, [maximum flow (youtube)](https://www.youtube.com/watch?v=ZBJHZi6Qu7s)* Student activity: Students to practise finding the minimum cut and maximum flow (flow capacity) for a range of networks.

Resource: maximum-flow-minimum-cut-sample-questions.DOCX |  |  |

Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.