 Year 12 Mathematics Standard 2

| MS-M6 Non-right-angled trigonometry | Unit duration |
| --- | --- |
| Measurement involves the application of knowledge, skills and understanding of numbers and geometry to quantify and solve problems in practical situations. Knowledge of measurement enables an understanding of basic daily situations involving rates and ratios, such as speed and the interpretation of maps and plans, effectively in a variety of situations. Study of measurement is important in developing students’ ability to solve problems related to two-dimensional and three-dimensional models and representations and to work effectively with a variety of rates and ratios. | 4 weeks |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is to solve problems involving right-angled and non-right-angled triangles in a variety of contexts. Students develop their ability to justify mathematical thinking and communicate solutions in an ordered and concise fashion. Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students. | A student:* interprets the results of measurements and calculations and makes judgements about their reasonableness, including the degree of accuracy and the conversion of units where appropriate MS2-12-3
* analyses two-dimensional and three-dimensional models to solve practical problems MS2-12-4
* chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times and methods for such use MS2-12-9
* uses mathematical argument and reasoning to evaluate conclusions, communicating a position clearly to others and justifying a response MS2-12-10

Related Life Skills outcomes: MALS6-3, MALS6-4, MALS6-13, MALS6-14 |

| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| Students will need to be familiar with Pythagoras’ theorem, the trigonometric ratios, and angles of elevation and depression from the Stage 5.1/5.2 Right-angled triangles (trigonometry) units. | During this unit, staff are encouraged to adopt formative assessment strategies to help develop mastery of skills. This may take the form of pre and post testing; exit slips; or mini-whiteboard activities that will inform practice. Challenging students to write sine or cosine rule questions will help identify when students know when to apply the rules. |

All outcomes referred to in this unit come from [Mathematics Standard Stage 6](https://syllabus.nesa.nsw.edu.au/mathematics-standard-stage6/) Syllabus © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2017

Glossary of terms

| Terms | Description |
| --- | --- |
| Ambiguous case | In trigonometry, the ambiguous case refers to using the sine rule to calculate the size of an angle in a triangle where there are two possibilities for the angle, one obtuse and one acute, leading to two possible triangles. |
| Angle of depression | When an observer looks at an object that is lower than 'the eye of the observer', the angle between the line of sight and the horizontal is called the angle of depression.Angle of depression diagram showing an eye on the left and a horizontal line from the eye towards the horizon. An object is shown below the horizontal line and a line, labelled the ‘line of sight’, extends from the eye down and to the object. The angle between the horizontal line to the line of sight is labelled ‘angle of depression’. |
| Angle of elevation | When an observer looks at an object that is higher than 'the eye of the observer', the angle between the line of sight and the horizontal is called the angle of elevation.Angle of elevation diagram showing an eye on the left and a horizontal line from the eye towards the horizon. An object is shown above the horizontal line and a line, labelled the ‘line of sight’, extends from the eye up and to the object. The angle from the horizontal line to the line of sight is labelled ‘angle of elevation’. |
| Compass bearing | Compass bearings are specified as angles either side of north or south. For example, a compass bearing of N500E is found by facing north and moving through an angle of 500 to the East. |
| Cosine rule | In any triangle ABC,$$c^{2}=a^{2}+b^{2}-2ab\cos(C) $$The image shows a scalene triangle ABC. AB is labelled c, BC is labelled a and AC is labelled b. |
| Radial survey | A radial survey can be used to measure the area of an irregular block of land. In a radial survey, a central point is chosen within the block of land and measurements are taken along intervals from this point to each vertex. The angles between these intervals at the central point are also measured and recorded. |
| Sine rule | In any triangle ABC,$$\frac{a}{sinA}=\frac{b}{sinB}=\frac{c}{sinC}$$The image shows a scalene triangle ABC. AB is labelled c, BC is labelled a and AC is labelled b. In words it says:Any side of a triangle over the sine of the opposite angle equals any other side of the triangle over the sine of its opposite angle. |
| True bearing | True bearings are measured in degrees clockwise from true north and are written with three digits being used to specify the direction.For example, the direction of north is specified 0000, east is specified as 0900, south is specified as 1800 and north-west is specified as 3150. |

| Lesson sequence | Content | Suggested teaching strategies and resources  | Date and initial | Comments, feedback, additional resources used |
| --- | --- | --- | --- | --- |
| Using right-angled trigonometric ratios to solve problems(1 or 2 lessons) | * review and use the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle **AAM** Paperclip icon
 | **Reviewing right-angled trigonometric ratios*** Students could be given a pre-test to determine the content that requires review
* Teacher to review the use of the sine, cosine and tangent ratios and are to show students how to use them to find the sides of a triangle. Consideration needs to be given to the different methods used when the variable is on the top of the ratio vs when the variable is on the bottom of the ratio. Students will also need to be shown how to find the angle for a given ratio.
* [Trigonometry pile up](http://www.greatmathsteachingideas.com/2012/03/12/trigonometry-pile-up/) is a great activity to practise the right-angled trigonometric ratios and Pythagoras’ theorem.
* Teachers could have different groups of students rounding off to different numbers of decimal places and then comparing their final answers to teach the importance of not rounding off too early.
* Students look at blood spatter to determine the angle of impact
* Teachers to read the article [bloodstain-pattern-analysis](https://science.howstuffworks.com/bloodstain-pattern-analysis3.htm) with students.
* Students work through the worksheet [blood spatter](https://www.yumpu.com/en/document/view/22869546/fsb-09-blood-spatter-worksheets-pdf-file-2632-kb/3)
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| Introducing the sine rule(2 or 3 lessons) | * use technology to investigate the sign of $\sin(A)$ and $\cos(A)$ for $0°\leq A\leq 180°$ Critical and creative thinking icon  Information and communication technology capability icon
* solve problems involving non-right-angled triangles using the sine rule, $\frac{a}{sin A}=\frac{b}{sin B}=\frac{c}{sin C}$ (ambiguous case excluded) **AAM** **Paperclip icon**
* find the size of an obtuse angle, given that it is obtuse
 | **Labelling a triangle*** All angles of the triangle are labelled using $A$, $B$ and $C$ (capital letters), generally labelled in an anti-clockwise direction starting from the top.
* All sides are labelled using $a$, $b$ and $c$ where $a$ is the opposite side to angle $A$, $b$ is the opposite side to angle $B$ etc.
* Changes to angle $A$ will result in a change to side $a$. They are linked. For the purposes of this unit the angle $A$ and side $a$ will be referred to as a matching pair. There are three matching pairs for each triangle.

**Investigating the sine and cosine ratios for obtuse angles*** Students should investigate the values of the sine and cosine ratios for all angles $A$, where $0\leq A\leq 180°$.
* Lead students to determine $\sin((180°-A))=\sin(A)$ and $\cos(\left(180°-A\right))=-\cos(A)$, where $0<A<90°$
* Discuss the ambiguity when $\sin(A)>0$, i.e. $A$ could be acute or obtuse when $\sin(A)$ is positive. There is no such ambiguity for the cosine ratio.

**Introducing the sine rule*** Teacher to introduce the rule $\frac{a}{sin A}=\frac{b}{sin B}=\frac{c}{sin C}$, where each part of the rule refers to a matching pair. Discuss its unusual characteristic of having two equals signs but only two parts are used each time to create a formula, i.e. $\frac{a}{sin A}=\frac{c}{sin C}$. This leads to three possible formulae being established from the rule. This rule is stated on the HSC Mathematics Standard 1 and 2 reference sheet.
* The sine rule stated above is presented in a form suitable for finding missing sides $a$, $b$ or $c$.
* Teachers need to explicitly discuss the conditions for using the sine rule, i.e. if the missing side or angle to be found is considered part of the information provided in the question, then having the information of two sides and two angles leads students to use the sine rule.
* Ideally the two sides and two angles refer to two matching pairs and this leads directly to using the sine rule.
* When the two sides and two angles are not matching pairs, the missing angle will need to be referenced using the information in the question to create matching pairs prior to using the sine rule.
* When finding missing angles, it is appropriate to use the reciprocal version of this rule, i.e. $\frac{\sin(A)}{a}=\frac{\sin(B)}{b}=\frac{\sin(C)}{c}$, and must use the inverse sine function, eventually, to determine the angle.
* For problems involving applications of the sine rule where an angle is to be found, any solution should only involve acute angles unless otherwise explicitly stated in the question.
* When using the sine rule to find an obtuse angle, students should apply the sine rule as usual and interpret the result using $\sin((180°-A))=\sin(A)$, which means subtracting the resulting angle from $180°$ to determine the obtuse angle.
* Students could complete a matching activity to learn the [sine rule](https://www.tes.com/teaching-resource/sine-rule-game-triginometry-6030231)
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| Introducing the cosine rule(2 lessons) | * solve problems involving non-right-angled triangles using the cosine rule, $c^{2}=a^{2}+b^{2}-2ab\cos(C)$ **AAM** **Paperclip icon**
 | **Introducing the cosine rule*** Teacher to introduce the cosine rule presented in two equivalent forms $ c^{2}=a^{2}+b^{2}-2ab cos C$ and $\cos(C)=\frac{a^{2}+b^{2}-c^{2}}{2ab}$. This rule (both forms) is stated on the HSC Mathematics Standard 1 and 2 reference sheet.
* Teachers need to explicitly discuss the conditions for using the cosine rule, i.e. if the missing side or angle to be found is considered part of the information provided in the question, then having the information of three sides and one angle leads students to use the cosine rule.
* The cosine rule is presented in two equivalent forms, one ($c$ is part of the subject) to determine the missing side length $c$ and the other ($C$ is part of the subject) to determine the missing angle $C$.
* It is possible to create the cosine rule where the missing side $a$ or missing angle $A$ is the subject, i.e. $a^{2}=b^{2}+c^{2}-2bc cos A$ and $\cos(A)=\frac{b^{2}+c^{2}-a^{2}}{2bc}$. The same can be applied for the missing side $b$ and missing angle $B$.

Note: Identifying the missing angle/side and labelling it as $C$/$c$ means that students can substitute directly into the formula on the reference sheet.* Students could complete this [impossible triangles](https://www.teachmathematics.net/page/19240/impossible-triangles) activity to review all of the rules they have learnt so far. Students need to come up with arguments as to whether a particular triangle can exist or not.
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| Practising solving a variety of triangle problems(3 lessons) | * solve practical problems involving Pythagoras’ theorem, the trigonometry of right-angled and non-right-angled triangles, angles of elevation and depression and the use of true bearings and compass bearings **AAM** **Paperclip icon** Critical and creative thinking icon
* work with angles correct to the nearest degree and/or minute
 | **Practising solving a variety of triangle problems*** Teacher to discuss with students how to round angles to the nearest degree or minute. Students’ attention will need to be drawn to the fact that the half-way point for rounding is 30 minutes or seconds.
* Students to use the [spider and fly problem](https://www.youtube.com/watch?v=tfHpsW6LBtE) to review Pythagoras’ theorem.
* Show students the start of the video and give them time to investigate before showing them the rest of the video which explains the solution. (Hint: students need to investigate the different ways they can create a net of the room)
* Students to investigate [using trig to measure the area of huge farms](http://education.abc.net.au/home#!/media/86350/)
* Watch the video on areas of huge farms
* Students to imagine mapping a triangular area of land on a quad bike. Ride directly north 2000 m then turn 60 degrees clockwise from north and ride in a straight line for 1500 m. At this point, follow a ridge straight back to your starting point. Draw this journey.
* Use the ridge as the hypotenuse and extend the first north distance to form a right-angled triangle. Use trigonometry (cosine function) and then Pythagoras' theorem ($a^{2} + b^{2} = c^{2}$) to calculate the distance you travelled along the ridge.
* Use ½ x base x height to calculate the area of your grazing land, which is the shape of an obtuse triangle. Hint: the height of this grazing land will be the distance you travelled east. Use sine function to work this out.
* Imagine you are aiming at a target 20 m away. If your aim could vary 5 degrees left or right, how much could you miss the target by? Hint: draw a diagram to form two right-angled triangles.
* Students can investigate the angle of the Leaning Tower of Pisa using the video [applying trigonometry - leaning tower](http://education.abc.net.au/home#!/media/154926/)
* Students watch the leaning tower video
* Draw the triangle made between the tower, the ground and the falling object. How far from the tower should the melons hit the ground? Use a trigonometric ratio to work out the distance to the nearest centimetre.
* Design a tower where the dropped object will fall exactly 100 m vertically and land exactly 10 m from the base of the tower. Include all measurements and angles.
* Students investigate: [are lasers accurate enough to track space debris?](http://education.abc.net.au/home#!/media/86394/)
* Watch video about lasers and space debris
* Let's say the radar beam had an accuracy of 1 degree and a target was 200 km away. Two right-angled triangles could be drawn, each with a base of 200 km and an angle of 0.5 degrees. The spread of the beam at 200 km would be: $2 x 200 tan(0.5) = 3.4907 km$. Laser beams may have an accuracy of 0.002 degrees. What would the spread of the laser beam be (in metres) at 200 km?
* How accurate are you? Find a wall and draw a target on it in chalk. Stand 20 m away and throw a wet tennis ball at the target. Measure the horizontal and vertical distances of spread. Use trigonometry to find your own degree of accuracy horizontally and vertically. Ask others to do the same, and compare your accuracy.
* Students could develop an orienteering course and/or complete an orienteering course around the school
* Students could measure the heights of buildings or other structures using a clinometer. Resource: [Make a Clinometer](https://www.wikihow.com/Make-a-Clinometer)
* The following resource includes solutions to exemplar questions provided by NESA

**Resource:** ms-m6-nesa-exemplar-question-solutions.DOCX |  |  |
| Finding the area of a non-right angled triangle(2 lessons) | * determine the area of any triangle, given two sides and an included angle, by using the rule $A=\frac{1}{2}absinC$, and solve related practical problems **AAM** **Paperclip icon**  Information and communication technology capability icon
* construct and interpret compass radial surveys and solve related problems  Information and communication technology capability icon Literacy icon Civics and citizenship icon
 | **Using the rule** $A=\frac{1}{2}absinC$ **to calculate area of non-right-angled triangles*** Teacher to introduce the rule for finding the area of a non-right angled triangle
* Student activity: Using [six maps](https://maps.six.nsw.gov.au/), have students zoom in and select an area (for example, irregular pentagon).
* Print the page and record the coordinates (use the coordinate tool).
* Choose a centre spot and draw a line north. (top of the page).
* Then draw lines to each vertex.
* Students then measure the angles and represent them as bearings.
* Students then use the cosine rule to find the area and perimeter of their shape.
* Students can use the distance and area tools in [six maps](https://maps.six.nsw.gov.au/) to check their calculations.
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Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All information and communication technologies (ICT), literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.