 Year 12 Mathematics Extension 2

Assessment task

MEX-V1 Further work with vectors

Driving question

Design an aerobatic display

Outcomes

* **MEX12-1** understands and uses different representations of numbers and functions to model, prove results and find solutions to problems in a variety of contexts
* **MEX12-3** uses vectors to model and solve problems in two and three dimensions
* **MEX12-7** applies various mathematical techniques and concepts to model and solve structured, unstructured and multi-step problems
* **MEX12-8** communicates and justifies abstract ideas and relationships using appropriate language, notation and logical argument

All outcomes referred to in this unit come from [Mathematics Extension 2](https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-2-2017) Syllabus © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2017

Learning across the curriculum

General capabilities

* Critical and creative thinking 
* Ethical understanding 
* Information and communication technology capability 
* Literacy 
* Numeracy 
* Work and enterprise 



[mcasyuma.marines.mil/Photos/igphoto/2001901282/](https://www.mcasyuma.marines.mil/Photos/igphoto/2001901282/)

Context

Students are to analyse the position and velocity of two stunt planes in space over a 10 second interval.

In this investigation, students will model the trajectory paths of the stunt planes using varying initial positions and velocity vectors for the stunt planes. Students will investigate the location, speed and direction of motion of the stunt planes and the distance they are apart.

In addition, students will need to determine and justify solutions to a range of scenarios, including the best position for a photographer to be placed to take the perfect action shot as well as considerations to flying distances between the planes.

Finally, students will need to design their own aerobatic display which satisfies a range of criteria.

For this problem the $z$ axis represents altitude.

Background

During this task students will

* Use parametric and vector representations to model motion in 3 dimensions
* Apply calculus and geometric results to analyse the motion
* Use reasoning and problem-solving skills
* Critically evaluate the modelling techniques explored in this assignment.

Task

Part 1: Develop the model

Consider two stunt planes, A and B, with the following trajectories:

* stunt plane A has initial position $(a,b,c)$ and velocity vector $\left[d,e,f\right]$ .
* stunt plane B has initial position $P\_{0}(x\_{0},y\_{0},z\_{0})$ at time $t=0 s$, a position $P\_{5}(x\_{5},y\_{5},z\_{5})$ at $t=5 s$ and a final position $P\_{10}(x\_{10},y\_{10},z\_{10})$ at $t=10s$

For some (sensibly) chosen values of $a, b, …e, f$ and three points $P\_{0}$, $P\_{5}$ and $P\_{10}$ find the parametric or vector equations for the position of each stunt plane at time *t*, where *t* is the time in seconds, using *metres* for the distance.

Use graphing software to simulate the motion of the stunt planes in 3 dimensions.

Part 2: Analyse using the model

Determine each of the following (if they occur at all):

* 1. When a plane is flying horizontal.
	2. When a plane is flying vertical.
	3. The position and speed of each stunt plane at time $t=3 s$
	4. When the stunt planes are flying parallel.
	5. When the stunt planes are flying perpendicular.
	6. The maximum speed of each stunt plane.
	7. The minimum speed of each stunt plane.
	8. When the planes paths intersect

Part 3: Apply the model

1. **Photography:**

A photographer is positioned so that their position looks directly onto the $xz$ plane and wishes to capture a range of perfect images. Recommend when the photographer should take each image and confirm any perfect image using graphing software.

* 1. When the stunt planes appear to intersect from the photographers perspective.
	2. When the stunt planes are flying parallel or as close to parallel from the photographers perspective.
	3. When the stunt planes are flying perpendicular or as close to perpendicular from the photographers perspective.
	4. When the difference in the stunt planes’ observed velocities are a minimum.
	5. When the difference in the stunt planes’ observed velocities are a maximum.
1. **Safety:**

The safety of pilots and crowds is considered when developing an aerobatic show.

* 1. A grandstand is to be placed in the $xy$ plane so that it is a minimum of 160m from the stunt planes at all times. Select and provide reasoning for a suitable location with consideration to the crowd’s perspective.
	2. When determining a flight path of planes flying in formation, it is important to consider the distance planes are apart.

	Fighter jet pilots can safely travel with their wing tips as close as 50cm apart.

	If at time t, stunt plane A is at position $P\_{t}$ and stunt plane B is at position $Q\_{t}$, establish the vector $\overset{\to }{P\_{t}Q\_{t}}$ and explain if the stunt planes satisfy this safety criteria.
1. **Pyrotechnics:**

A pyrotechnic device is to be placed on the ground and shoot a stream of fire for the stunt planes to fly through. The stream of fire has a maximum velocity of 10 metres per second and may be represented by a line in 3D.

Investigate a range of locations to place the pyrotechnic device and justify the best position and corresponding velocity vector so that both planes will fly through the stream of fire with consideration to the crowd in the grandstand’s perspective.

Part 4: Design an aerobatic display

A section of an aerobatic display is to be designed and modelled using vectors to represent each plane’s trajectory in three dimensions. The display must satisfy the following criteria:

* There are between 2 and 4 planes inclusive.
* The planes are to fly in formation such that the minimum distance between the planes is 50cm and the maximum distances is 50 metres.
* The planes must maintain an altitude of at least 150 metres.
* The display will last between 10 and 30 seconds.
* At least 1 stunt must be performed. Examples include a loop, hammerhead and Cuban eight or design another. A maximum of 3 stunts can be performed.
* The plane has the following airspeed limitations:
	+ Must never exceed 220 knots
	+ Stalls at 60 knots (must maintain an airspeed above this)

Students are to design their display and use appropriate graphical software and mathematical modelling techniques to justify that the criteria is satisfied.

What to submit

* Evidence of an authentic modelling. This may take the form of pictures of activities, screenshots of models with annotations.
* All initial conditions clearly determined.
* All formula, working and calculations required, either written by hand or typed. If screenshots have been provided, the formulas used need to be clearly annotated.
* All reasoning and justification, either written by hand or typed.

Success criteria

| Fluency, understanding and communication | Problem solving, reasoning and justification |
| --- | --- |

| Criteria | Working towards developing | Developing | Developed | Well developed | Highly developed |
| --- | --- | --- | --- | --- | --- |
| Part 1: Develop the model**(MEX12-1)** | Students choose values of a,b,c,d,e and f to determine the initial conditions. | Students develop vector or parametric equations to represent to position of each stunt plane at time t. | Students use graphing software to model or simulate the motion of the stunt planes. |  |  |
| Part 2: Analyse using the Model(MEX12-1, MEX12-3) | Students consider the direction and speed of travel of a stunt plane. | Students determine the maximum and minimum speed of a stunt plane. | Students compare the trajectory of the two stunt planes to determine if they are flying parallel and/or perpendicular. |  |  |
| **Part 3: Apply the Model: Photography**(MEX12-1, MEX12-3) |  | Students determine if the stunt planes appear to intersect from the observers’ perspective. | Students compare the trajectories of each plane to solved problems related to the direction of travel.  | Students compare the velocities of the two planes and reason when the differences are a minimum or maximum. |  |
| **Part 3: Apply the Model: Safety part a)****(MEX12-3, MEX12-7)** |  | A student determines a location for a grandstand that partially satisfies the distance criteria. | A student determines a location for a grandstand that satisfies the distance criteria. | A student determines the location of a grandstand, satisfying the distance criteria with consideration for the crowds perspective. |  |
| **Part 3: Apply the Model: Safety part b)****(MEX12-3, MEX12-7)** |  |  | A student determines the vector $\overset{\to }{P\_{t}Q\_{t}}$. | A students explains if the stunt plane satisfies the safety criteria.  |  |
| **Part 3: Apply the Model: Pyrotechnics****(MEX12-1, MEX12-3, MEX12-7, MEX12-8)** |  | Students determine the placement and velocity of the pyrotechnic device which satisfies most criteria. | Students determine the placement and velocity of the pyrotechnic device which satisfies the criteria. | Students determine the placement and velocity of the pyrotechnic device, satisfying the set criteria and supported by reasoning. | Students determine potential placements and velocities of the pyrotechnic device, satisfying the set criteria and supported by reasoning, and justifies the selection of a final position. |
| **Part 4: Design an aerobatic display****(MEX12-1, MEX12-3, MEX12-7, MEX12-8)** |  | Students are able to design an aerobatic display. | Students are able to design an aerobatic display that meets the criteria. | Students are able to design an aerobatic display that meets the criteria supported by mathematical reasoning. | Students are able to design an aerobatic display that meets the criteria justified with graphical software and mathematical modelling techniques. |

Note**s**

* Any non-attempt in a section will be deemed zero. Marks can only be attributed to attempted responses.
* Corresponding question numbers are shown in brackets.

Note to staff

The success criteria above has been designed for students and staff alike to use. Students should be presented the rubric as part of the assessment task package. Students and staff follow the process of the task downwards through the rubric and the depth of responses, for each element, across the rubric. Students should be encouraged to use the rubric to self-assess their progress as an assessment-as-learning strategy.

The aim of the assessment task is to develop students’ deep content knowledge. This is reflected in the descriptors, **working towards developing** through to **highly developed**. The level of skill and understanding required in each part of the task is different; some parts require **highly developed** or **well-developed** skills, other parts only capture a **developing** skill set.

None of the working mathematically elements are distinct and when demonstrating one element, you are invariably demonstrating another. As an example, communication runs concurrently through all the other working mathematically elements. Students cannot respond to this assessment without communicating in some form. However, it is envisaged that there is a general progression through the working mathematically elements, starting with fluency and leading to understanding, problem solving, reasoning and justification, with increasingly higher levels of communication accompanying each element. Careful consideration has been given to the position of the success criteria statements so they reflect the working mathematically elements demonstrated.

This assessment task has been designed to illuminate the style of questions and the types of responses needed to elicit deep content knowledge, however, staff are encouraged to use and adapt the assessment task and the success criteria to their school context. Staff may like to enhance or amend sections of the task. Staff may like to adapt the rubric to assign marks to the descriptors in order to differentiate between responses that address the same statement. All changes are the responsibility of the staff using the assessment.