 Quadratic equations

Quadratic equations with complex coefficients

1. $ix^{2}-i=0$

$$i\left(x^{2}-1\right)=0$$

$$i\left(x-1\right)(x+1)=0$$

$$x=\pm 1$$

1. $ix^{2}-x+12i=0$

$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$

Substitute $a=i, b=-1 and c=12i$

$$x=\frac{-(-1)\pm \sqrt{(-1)^{2}-4×i×12i}}{2×i}$$

$$=\frac{1\pm \sqrt{1+48}}{2i}$$

$$=\frac{1\pm \sqrt{49}}{2i}$$

$$=\frac{1\pm 7}{2i}$$

$x=\frac{8}{2i}$ or $x=\frac{-6}{2i}$

$x=\frac{4}{i}$ or $x=\frac{-3}{i}$

Multiply the numerator and denominator by $i$

$x=\frac{4i}{-1}$ or $x=\frac{-3i}{-1}$

$x=-4i$ or $x=3i$ [all solutions must be presented as a complex number in the form $a+bi$]

1. $x^{2}-4\left(1+i\right)x+10i=0$

$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$

Substitute $a=1, b=-4-4i and c=10i$

$$x=\frac{-(-4-4i)\pm \sqrt{(-4-4i)^{2}-4×1×10i}}{2×1}$$

$$=\frac{4+4i\pm \sqrt{(16+32i-16)-40i}}{2}$$

$$=\frac{4+4i\pm \sqrt{32i-40i}}{2}$$

$$=\frac{4+4i\pm \sqrt{-8i}}{2}$$

$$=\frac{4+4i\pm 2\sqrt{-2i}}{2}$$

$$=2+2i\pm \sqrt{-2i}$$

Evaluate $\sqrt{-2i}$

$$a+bi=\sqrt{-2i}$$

$$\left(a+bi\right)^{2}=-2i$$

$$a^{2}+2abi-b^{2}=-2i$$

$$a^{2}-b^{2}+2abi=-2i$$

Equating the coefficients:

$$a^{2}-b^{2}=0 $$

$$2ab=-2$$

$$ab=-1$$

By inspection

$$a=1, b=-1 or a=-1, b=1 $$

$$\sqrt{-2i}=1-i or \sqrt{-2i}=-1+i$$

$$x=2+2i\pm \sqrt{-2i}$$

$x=2+2i\pm (1-i)$ or $x=2+2i\pm (-1+i)$

$x=2+2i+(1-i)$ or $x=2+2i-\left(1-i\right)$ or $x=2+2i+\left(-1+i\right)$ or $x=2+2i-(-1+i)$

$x=2+2i+1-i$ or $x=2+2i-1+i$ or $x=2+2i-1+i$ or $x=2+2i+1-i$

$x=3+i$ or $x=1+3i$ or $x=1+3i$ or $x=3+i$

Observe that the solutions are repeated.

$x=3+i$ or $x=1+3i$