 Identities involving modulus and argument

Identity 1

1. and

**Part 1:**

Let and

and

**Part 2:**

Let and

From Part 1:

Identity 2

1. **and ,**

**Part 1:**

Let and

and

Realise the denominator

Factorise the numerator by grouping in pairs.

**Part 2:**

Let and

From Part 1:

Identity 3

1. and

**Method 1: Using the 1st and 2nd identities:**

(n times)

(n times)

(n times)

(n times)

**Method 2: Mathematical Induction**

**Part 1:**

Step 1: Show true for .

True for .

Step 2: Assume true for where is a positive even integer. i.e. Assume

Step 3: Show true for . i.e. Show .

From step 2 (assumption)

Step 4: Conclusion.

**Part 2:**

Let

Step 1: Show true for .

True for .

Step 2: Assume true for where is a positive even integer. i.e.

Step 3: Show true for . i.e. Show .

From step 2 (assumption)

Step 4: Conclusion.

**Method 3: Exponential form**

Let be a complex number.

In exponential form, , is the modulus of and is the argument of

, is the modulus of and is the argument of

Identity 4

1. and ,

**Method 1: Using the 1st and 2nd identities:**

(n times)

(n times)

(n times)

(n times)

(n times)

(n times)

(n times)

(n times)

**Method 2: Mathematical Induction**

**Part 1:**

Step 1: Show true for .

True for .

Step 2: Assume true for where is a positive even integer. I.e. Assume

Step 3: Show true for . i.e. Show

From step 2 (assumption)

Step 4: Conclusion.

**Part 2:**

Let

Step 1: Show true for .

True for .

Step 2: Assume true for where is a positive even integer. i.e.

Step 3: Show true for . i.e. Show

From step 2 (assumption) and from step 1

Step 4: Conclusion.

**Method 3: Exponential form**

Let be a complex number.

In exponential form, , is the modulus of and is the argument of

, is the modulus of and is the argument of

Identity 5

Let and

Identity 6

Let and

Identity 7

If then

Identity 8

If then

Identity 9

If then