 Year 12 Mathematics Extension 2

| MEX-M1 Applications of calculus to mechanics | Unit duration |
| --- | --- |
| The topic Mechanics involves the study of change in the motion of objects when acted upon by forces. It involves the mathematical representation of quantities with magnitude and direction and their representation graphically and algebraically.A knowledge of mechanics enables understanding of the behaviour of objects according to mathematical law in order to model physical systems and predict the behaviour of objects that are under the influence of forces such as gravity and air resistance.The study of mechanics is important in developing students’ understanding of changes in motion, modelling change situations involving a variety of mathematical techniques and contexts and using algebraic and graphical techniques to describe and solve problems and to predict future outcomes with relevance to, for example Physics. | 7 weeks |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is to model the mechanics of objects in a variety of situations, with and without resistance. Models of systems are developed and used to predict the behaviour of objects under the influence of forces such as gravity or air resistance.Students develop experience in applying calculus techniques to the solution of a range of physical problems. Applications require the use of techniques from other sections of the course and students have the opportunity to develop high-level problem-solving skills. The connections between mathematical representations and physical descriptions of motion are an essential part of Applied Mathematics. | A student:* uses mechanics to model and solve practical problems MEX12-6
* applies various mathematical techniques and concepts to model and solve structured, unstructured and multi-step problems MEX12-7
* communicates and justifies abstract ideas and relationships using appropriate language, notation and logical argument MEX12-8
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| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| Students should have studied the topics of MEX-C1 Further integration, MEX-V1 Further work with Vectors and the Year 12 Mathematics Extension 1 topic of ME-V1 Introduction to Vectors. | * Staff should adopt formative assessment strategies to support the mastery of the key skills in this topic, such as the identification and application of appropriate identities for acceleration when integrating or the construction of forces diagrams and resolving forces to form equations. Staff should adopt what-if style questions and spark debate regarding scenarios involving motion to gauge students’ conceptual understanding.
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All outcomes referred to in this unit come from [Mathematics Extension 2](http://www.educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-2-2017) Syllabus
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Glossary of terms

| Term | Description |
| --- | --- |
| Inelastic | Inelastic describes a string or rod that cannot be extended or stretched. |
| Resisted motion | Resisted motion is motion that encounters resisting forces, for example friction and air resistance. |
| Smooth | When an objects is in contact with a smooth surface, the frictional forces between them are negligible or zero. |
| Terminal velocity | Terminal velocity is the constant velocity that a free-falling object will eventually reach when the resistance of the medium through which the object is falling prevents further acceleration. |

| **Lesson sequence** | **Content** | **Suggested teaching strategies and resources**  | **Date and initial** | **Comments, feedback, additional resources used** |
| --- | --- | --- | --- | --- |
|  |  | **Kinematics:** The study of the motion of a body without reference to the causes of its motion.**Mechanics:** The study of the effect of forces on a body to cause them to move or to alter their state of motion. Since forces produce acceleration, principles of kinematics are used. All bodies are regarded as particles. |  |  |
| Introducing simple harmonic motion(1 to 2 lessons) | **M1.1: Simple harmonic motion*** derive equations for displacement, velocity and acceleration in terms of time, given that a motion is simple harmonic and describe the motion modelled by these equations **AAM**
	+ establish that simple harmonic motion is modelled by equations of the form:

 or , where is displacement from a fixed point, is the amplitude, is the period, is the phase shift and is the central point of motion* + establish that when a particle moves in simple harmonic motion about , the central point of motion, then
* prove that motion is simple harmonic when given an equation of motion for acceleration, velocity or displacement and describe the resulting motion
 | **Introducing simple harmonic motion (SHM)*** This topic used to be part of the Mathematics Extension 1 course, prior to 2018. Therefore resources for this course may be appropriate for this section.
* No reference to forces are needed for the SHM section.

Introduce the concept of simple harmonic motion by:* Running the SHM activity. **Resource:** mex-m1-simple-harmonic-motion-activity.DOCX
* Examining the SHM of a spring [Geogebra interactive activity](https://www.geogebra.org/classic/Ne3TwAdW).
* Examining the SHM of a Ferris wheel [Geogebra interactive activity](https://www.geogebra.org/m/qyfesjMV).

Establish equations of motion for SHM* Starting with a model for displacement verses time in the form use differentiation techniques to determine models for velocity over time and acceleration over time. Express acceleration in the form
* Repeat this process with a model for displacement over time in the form and ask students to determine a model for acceleration in terms of
* Discuss the appropriateness for either model by referencing the initial displacement of the particle. Students establish that if at the particle is at the centre of the SHM, then a sine model is more appropriate, whereas if the particle is at an extremity at , then a cosine model is more appropriate.
* Establish that is the amplitude and is the period of the motion.

Translations of the SHM model* Introduce vertical translations of the model in the form and , such that the simple harmonic motion is centred about a displacement . Lead students to:
	+ determine models for velocity and acceleration in terms of time, .
	+ express acceleration as a function of displacement in the form
* Introduce horizontal translations, or lag or phase shift, of the model in the form and Lead students to:
	+ determine models for velocity and acceleration in terms of time, .
	+ express acceleration as a function of displacement in the form
* Introduce composite vertical and horizontal translations of the model in the form and . Lead students to:
	+ determine models for velocity and acceleration in terms of time, t.
	+ express acceleration as a function of displacement in the form

Proving motion is simple harmonic* Students prove that an equation of motion for acceleration, velocity or displacement is simple harmonic by showing .
* Students describe the motion in terms of its amplitude, period, phase shift and central point of motion.

The [Australian Mathematical Sciences Institute AMSI website](http://amsi.org.au/ESA_Senior_Years/SeniorTopic3/3_md/SeniorTopic3i.html#links_1) has a resource booklet (PDF document) for motion in a straight line, including SHM. |  |  |
| Interpreting graphs of SHM(1 lesson) | * sketch graphs of and as functions of and interpret and describe features of the motion
* prove that motion is simple harmonic when given graphs of motion for acceleration, velocity or displacement and determine equations for the motion and describe the resulting motion
 | **Interpreting graphs of SHM*** The use of interactive applets can be used to assist students in interpreting the graphs of motion.
	+ SHM of a spring [Geogebra interactive activity](https://www.geogebra.org/classic/Ne3TwAdW).
	+ SHM of a Ferris wheel [Geogebra interactive activity](https://www.geogebra.org/m/qyfesjMV).
* Interpreting the velocity and acceleration of the motion
	+ Lead students to determine that a particle is accelerating if velocity and acceleration are acting in the same direction, i.e. if the first and second derivatives are the same sign – both positive or both negative
	+ Lead students to determine that a particle is decelerating if acceleration is opposing velocity, i.e. if the first and second derivatives are different signs.
	+ Link acceleration and deceleration to their experiences in the SHM activity used to introduce this concept.
	+ Determine the displacement when velocity is a minimum or a maximum, as well as the displacement when acceleration is a minimum or a maximum.

Student activities:* Given a **function** of displacement, velocity or acceleration, students:
	+ find equations for and as functions of
	+ sketch graphs of and as functions of
	+ interpret and describe features of the motion
* Given a **graph** of displacement, velocity or acceleration, students:
	+ find equations for and as functions of
	+ determine if the motion is simple harmonic
	+ describe the motion
 |  |  |
| Determining the speed during SHM(1 lesson) | * derive and the equations for velocity and displacement in terms of time when given and initial conditions, and describe the resulting motion
 | **Determining speed during SHM centred at zero*** Derive given
	+ Starting with use the identity to determine and
	+ When is moving with SHM, and when , where is the amplitude.
	+ Therefore with conditions when
	+ Lead students to determine that
	+ Use this result to determine the speed of the particle, as speed , therefore

**Determining speed during SHM centred at** * Derive given
	+ Start with and to determine with conditions when or
	+ Lead students to determine that
	+ Use this result to determine the speed of the particle, as speed , therefore

**Derive equations for velocity and displacement in terms of time given :*** Determine the appropriate equation by considering the initial conditions. i.e. or
* Using the initial conditions and the equation of acceleration to find and
* Determine the equation for displacement in terms of time.
* Determine the equation for velocity in terms of time.

**Sample examination question:** A particle is moving along the x-axis in simple harmonic motion. The displacement of the particle is metres. The particle is at rest when and when . It takes 6 seconds to travel from to . What is the maximum speed of the particle?* 1. ms-1
	2. ms-1
	3. ms-1
	4. ms-1

**Resource:** mex-m1-sample-questions.DOCX |  |  |
| Solving Problems with SHM(1 lesson) | * use relevant formulae and graphs to solve problems involving simple harmonic motion **AAM** Critical and creative thinking icon Information and communication technology capability icon
 | **Solving Problems with SHM**[**2016 HSC Extension 1**](https://www.educationstandards.nsw.edu.au/wps/wcm/connect/756b83c4-d51a-4033-ae36-ff7cd39d4369/2016-hsc-maths-ext-1.pdf?MOD=AJPERES&CACHEID=ROOTWORKSPACE-756b83c4-d51a-4033-ae36-ff7cd39d4369-lCkVKC5) **Mathematics Question 13 a**The tide can be modelled using simple harmonic motion. At a particular location, the high tide is metres and the low tide is metre. At this location the tide completes 2 full periods every hours. Let be the time in hours after the first high tide today.* Explain why the tide can be modelled by the function
* The first high tide tomorrow is at 2 am. What is the earliest time tomorrow at which the tide is increasing at the fastest rate?

[**2004 HSC Extension 1**](https://www.educationstandards.nsw.edu.au/wps/wcm/connect/b304d893-e234-4c76-9bec-6d16d1b365cb/maths-ext1-hsc-exam-2004.pdf?MOD=AJPERES&CACHEID=ROOTWORKSPACE-b304d893-e234-4c76-9bec-6d16d1b365cb-lGd7Yut) **Mathematics Question 3**A ferry wharf consists of a floating pontoon linked to a jetty by a metre long walkway. Let metres be the difference in height between the top of the pontoon and the top of the jetty and let metres be the horizontal distance between the pontoon and the jetty.Diagram showing the pontoon, jetty and the walkway. * Find an expression for in terms of .
* When the top of the pontoon is 1 metre lower than the top of the jetty, the tide is rising at a rate of 0.3 metres per hour. At what rate is the pontoon moving away from the jetty?

**HSC Extension 2 Mathematics syllabus prior to 2018** The deck of a ship was below the level of a wharf at low tide and above wharf level at high tide. Low tide was at 8.30 am and high tide at 2.35 pm. Find when the deck was level with the wharf, if the motion of the tide was simple harmonic. |  |  |
| Introduction to mechanics(2 lessons) | **M1.2: Modelling motion without resistance*** examine force, acceleration, action and reaction under constant and non-constant force (ACMSM133, ACMSM134) **AAM** Critical and creative thinking icon  Information and communication technology capability icon
* examine motion of a body under concurrent forces (ACMSM135) **AAM** Critical and creative thinking icon  Information and communication technology capability icon
 | **Defining types of forces*** Weight (or gravitational)
	+ A force that pulls objects towards the centre of the earth with an acceleration
	+ Objects under the force of weight are represented as a particle (a point at the centre of mass)
* Resistive
	+ Particles moving in a medium (liquid or gas) are subjected to a force that resists movement.
	+ The force acts in the opposite direction to the motion.
* Frictional
	+ A type of resistive force that is generated when the surfaces of solid objects come into contact with each other.
	+ Friction opposes motion.
* Normal
	+ This is a type of reactive force that is exerted when an object applies a force on another object.
	+ It acts perpendicular to the surfaces at the point of contact (hence its name)
* Tensional
	+ This is a type of reactive force that acts in a string or rod.
	+ It is a pull force that acts along the string or rod.
	+ The magnitude of the tension is constant throughout the string or rod.
	+ The tension force acts in opposite directions at opposite ends of the string or rod.
* Centripetal
	+ Centripetal describes a resultant force that causes an object to move in a circular path.
	+ It always acts towards the centre of the circular path.
	+ **This course does not cover circular motion.**

**Newtons (three) laws of motion*** Introduce Newton’s laws of motion
	+ A body remains in a state of rest or uniform motion in a straight line unless acted upon by a non-zero resultant force.
	+ The rate of change of momentum is proportional to the applied resultant force and occurs in the direction of the force.
	+ The forces of action and reaction between contacting bodies are equal in magnitude and opposite in direction.
* Newton’s first law of motion: Discuss inertia as tendency to remain in a state of constant or zero velocity. Newton’s first law implies inertia can only be changed if acted upon by a (resultant) force.
* Newton’s second law of motion: This law describes how motion is affected by an external force and that , where is the force vector, is the mass of the object and is the acceleration of the object as a result of the force. It may be useful to define momentum here - momentum of an object is equal to the product of the objects mass and its velocity.
* Newton’s third law of motion: This law determines how forces on objects interact with each other. If an object A exerts a force on object B, then object B also exerts a force on object A which is equal in magnitude but in the opposite direction.

**Generating forces (at a point) diagrams*** For the purposes of solving problems involving forces it is imperative to generate a situational diagram (a quick sketch which describes the context from the problem) and a forces diagram.
* Any given problem may require more than one forces diagram. A forces diagram is a simple vectors diagram which describes the forces acting on a particle. The forces are represented as arrows indicating the direction of the forces with angles (if possible) to horizontal and vertical axes. (Sometimes it is useful to provide angles to axes perpendicular to surfaces). The magnitude of the forces is represented, simply, as quantities next to the arrows.
* The *Forces Diagrams* resource from the [Mechanics – Forces and Newton’s laws of motion](http://www.mathcentre.ac.uk/topics/mechanics/newton/) section (under quick references) of Mathcentre.co.uk provides instructions for constructing forces diagrams.

**Resolving forces orthogonally*** When analysing situations involving forces converging at a point from different directions, it is simpler to resolve the forces orthogonally to determine how they interact. Forces are generally resolved along the and axes, although they can be resolved in any directions perpendicular to each other.
* Representing forces in component form or as column vectors will be useful here.
* The *Equilibrium of a Particle* resource from the [Introduction to Mechanic section](http://www.mathcentre.ac.uk/topics/mechanics/intro/) (under quick references) of Mathcentre.co.uk provides instructions for resolving forces.

**Resultant Forces*** Forces meeting at a point can be represented as a net force called the resultant force. The resultant force is calculated as the vector sum of the forces meeting at a point, i.e. if forces , and act on a point, the resultant force or in more general terms
* Forces are in equilibrium, a stationary state or state of constant velocity, if the resultant force is equal to zero.
* Applying Newton’s second law of motion to a non-zero resultant force results in a change of motion determined by

where is the acceleration of the particle in the direction of the resultant force.**Lami’s theorem*** Although Lami’s theorem is not explicitly referenced in the syllabus, it should be considered as it offers quick and simple methods for solving problems when three forces are in equilibrium
* Lami’s theorem:

The left diagram shows three forces at a point. The right diagram demonstrates how Lami's theorem can be used to construct a tip-to-tail triangle of forces.Each of the forces needs to be translated so that they form a tip-to-tail triangle of forces, as shown.Applying the sine rule gives and  (Lami’s Theorem)Similarly the cosine rule gives* Consider three forces , and acting on a point, in equilibrium. If the force is removed, the resultant force generated from and is equal to .
* The following [Geogebra activity](https://www.geogebra.org/m/dprezdz6) demonstrates Lami’s theorem in action.

**VCE Specialist Mathematics questions*** [VCE Specialist Mathematics exam past papers](https://www.vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx)
* [2018 VCE Specialist Mathematics examination 2, Section A Question 16](https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018specmath2-w.pdf)
* [2018 VCE Specialist Mathematics examination 1, Question 1](https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018specmath1-w.pdf)
* [2017 VCE Specialist Mathematics examination 2, Section A Question 17](https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017specmath2-w.pdf)
* [2016 VCE Specialist Mathematics examination 2, Section A Question 14](https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016specmath2-w.pdf)
 |  |  |
| Determining and applying equations of motion(1 or 2 lessons) | * consider and solve problems involving motion in a straight line with both constant and non-constant acceleration and derive and use the expressions , and for acceleration (ACMSM136) **AAM** Critical and creative thinking icon
* use Newton’s laws to obtain equations of motion in situations involving motion other than projectile motion or simple harmonic motion **AAM**
	+ use where is the force acting on a mass, , with acceleration
* describe mathematically the motion of particles in situations other than projectile motion and simple harmonic motion **AAM**
	+ interpret graphs of displacement-time and velocity-time to describe the motion of a particle, including the possible direction of a force which acts on the particle
* derive and use the equations of motion of a particle travelling in a straight line with both constant and variable acceleration (ACMSM114) **AAM**
 | **Equations of motion for constant acceleration (Newton’s equations of motion)*** Building on students’ understanding of the previous lessons, students need to determine equations where acceleration is the subject, by examining the resultant force, and use integration techniques to determine velocity and displacement equations
* For situations involving constant forces and constant acceleration, links need to be established with Newton’s (four) equations of motion (below). Students cannot reference these equations within a free response but they will provide efficient strategies for answering multiple choice questions

where is the displacement, is the initial velocity, is the final velocity, is the constant acceleration and is time. Note: In Stage 6 Physics, is used for displacement.**VCE Specialist Mathematics questions*** [VCE Specialist Mathematics exam past papers](https://www.vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx)
* [2018 VCE Specialist Mathematics examination 2, Section A Question 15](https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018specmath2-w.pdf)
* [2017 VCE Specialist Mathematics examination 1, Question 9](https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017specmath1-w.pdf)

**Equations of motion for non-constant acceleration*** Derive the expressions for non-constant acceleration:

 (express using the chain rule)  (as required) differentiate both sides with respect to )* Students use the expressions , and for acceleration , as such:
	+ If use to form the equation and integrate with respect to
	+ If use to form the equation

, manipulate the equation to form and integrate with respect to [this is a particular focus for the resisted motion sub-topic]* + If use to form the equation and integrate with respect to

Note:* + If let to derive an equation for displacement as a function of time.
	+ If let to derive an equation for velocity as a function of time.

**VCE Specialist Mathematics questions*** [VCE Specialist Mathematics exam past papers](https://www.vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx)
* [2016 VCE Specialist Mathematics examination 2, Section A Question 15](https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016specmath2-w.pdf)
 |  |  |
| Analysing motion through a resisting medium(1 lesson) | **M1.3: Resisted motion*** solve problems involving resisted motion of a particle moving along a horizontal line **AAM** Critical and creative thinking icon
	+ derive, from Newton’s laws of motion, the equation of motion of a particle moving in a single direction under a resistance proportional to a power of the speed
	+ derive an expression for velocity as a function of time
	+ derive an expression for velocity as a function of displacement
	+ derive an expression for displacement as a function of time
	+ solve problems involving resisted motion along a horizontal line
 | **Introducing resisted motion and the idea of drag** * Introduce this topic by discussing how it feels to be running in a direction following wind (assistance) and against wind (resistance).
* Consider this scenario: [On May 31, 2008, Usain Bolt](https://www.youtube.com/watch?time_continue=3&v=oVKHYOyWo1c) (duration 3:26) ran a time of seconds with a wind, officially breaking the 100 meter world record. On [June 29, 2008, Tyson Gay](https://www.youtube.com/watch?v=K9Da1Womuk0) (duration 5:14) ran a time of seconds with wind. Despite having a faster time, he did not break the world record. Discuss why 100m world record attempts will only be registered with a maximum wind assistance of .
* Discuss car, motorbike, bicycle, aeroplane and supermaxi yacht racing and the effect of drag and how aerodynamic designs or techniques aim to minimise drag, e.g. motorbike riders tuck into a ball-like position during straights to increase speed, while opening up their bodies to increase braking on approaches to corners. Bicycle riders adopt strategies of riding behind another rider or forming triangle-like formations to [reduce the drag on the lead rider](https://www.wired.com/story/the-physics-of-drafting-in-the-tour-de-france/).
* Discuss other types of mediums, other than air, like water or even more viscous mediums like honey.
* Lead discussions around factors that influence resistance including surface area and speed. Discuss the notion that an object colliding with gaseous/liquid particles will resist motion and the more collisions the greater the impact of resistance.
* Situations involving resisted motion will describe an object rather than a particle, as a particle implies an infinitely small surface area leading to no resistance force.

Sample question:* An object is moving on a smooth horizontal plane at in a resisted medium. The resistance force acting on the particle is proportional to the square of the speed and is the only force acting on the particle throughout the duration of its motion. After metres, the particle moves at .
	1. Show that the model for velocity in terms of displacement is equal to
	2. Calculate the velocity of the object after 12 metres.
	3. Develop a model for displacement as a function of time, describing the motion of the particle.
	4. Develop a model for velocity as a function of time.
	5. State the limitations of the model
 |  |  |
| Solving problems involving motion upwards and downwards in a resisting medium(2 lessons) | * solve problems involving the motion of a particle moving vertically (upwards or downwards) in a resisting medium and under the influence of gravity **AAM** Critical and creative thinking icon Information and communication technology capability icon
	+ derive, from Newton’s laws of motion, the equation of motion of a particle moving vertically in a medium, with a resistance proportional to the first or second power of its speed
	+ derive an expression for velocity as a function of time and for velocity as a function of displacement (or vice versa)
	+ derive an expression for displacement as a function of time
	+ determine the terminal velocity of a falling particle from its equation of motion
	+ solve problems by using the expressions derived for acceleration, velocity and displacement including but not limited to obtaining the maximum height reached by a particle, and the time taken to reach this maximum height and obtaining the time taken for a particle to reach ground level when falling
 | **Analysing motion of an object falling in a resisted medium*** Build on students understanding from horizontal motion under resistance but with the acknowledgement of weight (gravitational force).
* Students need to be explicitly taught to draw a forces diagram prior to developing an equation of motion for acceleration.
* Students should define the direction of motion (downwards) as positive to simplify the equations and interpretation of their results.
* Consider an object falling under resistance proportional to the square of its speed:

Diagram showing the forces acting on a falling object where resistance is proportional to the square of its speed.* By inspecting the resultant force, and
* Students will need to apply and manipulate the expressionto determine the equations of motion. i.e. forming the integral
* Lead students to recognise this integral can be manipulated into the form ,

i.e. [**2018 HSC Extension 2**](https://www.educationstandards.nsw.edu.au/wps/wcm/connect/faaffce8-847b-4fc6-8d72-4cb072dd8c56/2018-hsc-mathematics-ext-2.pdf?MOD=AJPERES&CACHEID=ROOTWORKSPACE-faaffce8-847b-4fc6-8d72-4cb072dd8c56-mr2.1-X) **Mathematics Question 14b*** A falling particle experiences forces due to gravity and air resistance. The acceleration of the particle is , where and are positive constants and is the speed of the particle. (Do NOT prove this.)

Prove that, after falling from rest through a distance, , the speed of the particle .**Analysing motion of an object fired vertically upwards in a resisted medium*** Lead students to determine that the motion in the upward phase of motion must be modelled differently and distinctly from the downwards phase.
* Similar to motion in a downwards direction, students need to generate a forces diagram with the direction of motion (upwards) defined as positive.
* Lead students to determine an equation of motion for acceleration by using the identity.
* Students will need to manipulate the equation to form an integral in standard form to develop the other equations of motion.
* Lead students to determine the conditions for terminal velocity – the velocity when the resistance force is equal to the weight of the object.
* Challenge students to consider whether it takes the same amount of time for an object to travel to a maximum height of metres after being fired upwards, as it does to fall back metres to its original starting point. Lead students to the conclusion that it will take longer on the downwards part of the journey as the magnitude of acceleration downwards is less than the magnitude of deceleration upwards.

[**HSC Mathematics Extension 2**](https://www.educationstandards.nsw.edu.au/wps/portal/nesa/resource-finder/hsc-exam-papers/2018/mathematics-extension-2-2018-hsc-exam-pack) **past papers*** [HSC 2013](https://www.educationstandards.nsw.edu.au/wps/wcm/connect/638dc23a-bf2b-414c-9062-514c721320d3/maths-ext2-hsc-exam-2013.pdf?MOD=AJPERES&CACHEID=ROOTWORKSPACE-638dc23a-bf2b-414c-9062-514c721320d3-lG9BwCj) Question 15d
* [HSC 2014](https://www.educationstandards.nsw.edu.au/wps/wcm/connect/2fe9d8cc-da19-4400-992d-ada08c778998/maths-ext2-hsc-exam-2014.pdf?MOD=AJPERES&CACHEID=ROOTWORKSPACE-2fe9d8cc-da19-4400-992d-ada08c778998-lG96Dba) Question 14c
 |  |  |
| Solving problems with projectiles in resisted motion(1 or 2 lessons) | **M1.4: Projectiles and resisted motion*** solve problems involving projectiles in a variety of contexts AAM Critical and creative thinking icon
	+ use parametric equations of a projectile to determine a corresponding Cartesian equation for the projectile  Information and communication technology capability icon
	+ use the Cartesian equation of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown
* solve problems involving projectile motion in a resisting medium and under the influence of gravity which include consideration of the complete motion of a particle projected vertically upwards or at an angle to the horizontal **AAM** Critical and creative thinking icon Information and communication technology capability icon
 | Solving problems with projectiles in resisted motion* Students will need to tie together their understanding of resisted motion in horizontal, upwards and downwards directions in this one scenario. Students need to clearly define these stages through projectile motion under resistance.
* Consider an object fired from ground level at at an angle of with air resistance proportional to the square of the velocity.
* Draw the scenario

Diagram showing the trajectory of an object split into the upwards and downwards trajectory. The initial velocity is shown as 40 metres per second at an angle of 35 degrees to the horizontal.* Draw (three) forces diagrams representing the forces and motion in a horizontal, upwards and downwards direction. Define the direction of motion as positive in each case.

Diagram showing the forces acting on a trajectory where air resistance is proportional to the square of the velocity. When travelling upwards, air resistance works with the force of gravity, when travelling downwards, air resistance opposes the force of gravity.* Form parametric equations describing horizontal, upwards and downwards motion. Use and (or similar notation)to representthe acceleration upwards and downwards respectively.
	+ *→*
	+ *→*
	+ *→*
* Adopt integration techniques including the use of the identities expressions , and for acceleration to form equations of motion using the component velocities at time
 |  |  |

Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.