 ME-T3 unit worked solutions

Lesson: Converting expressions to and

Example 1: Show

Let

Expanding the right hand side gives,

Equating the coefficients gives,

 and

Since sine and cosine are positive, must be a 1st quadrant angle.

Calculate by forming a right angle triangle and applying Pythagoras’ theorem or using the trigonometric identity

**Method 1:**



**Method 2:**

By squaring and adding the equations and we obtain

Calculate , noting that because sine and cosine are positive, must be a first quadrant angle.

Example 2: Find 4 expressions equivalent to

Expression 1

From the worked example above,

Expression 2

Let

Expand the right hand side:

Equate the coefficients: and or

Since sine is negative and cosine is positive, must be a 4th quadrant angle.

Find by forming a right angle triangle in the 4th quadrant and applying Pythagoras’ theorem, or using the trigonometric identity Obtain .

Find

This second expression follows logically from the first since . It represents a phase shift of radians.

Expression 3

Let

Expand the right hand side:

Equate the coefficients: and

Since sine and cosine are positive, must be a 1st quadrant angle.

Find by forming a right angle triangle in the 1st quadrant and applying Pythagoras’ theorem or using the trigonometric identity Obtain .

Find

Expression 4

Let

Expand the right hand side:

Equate the coefficients: and

Since sine is negative and cosine is positive, must be a 4th quadrant angle.

Find by forming a right angle triangle in the 4th quadrant and applying Pythagoras’ theorem or using the trigonometric identity Obtain .

Find

This second expression follows logically from the third since . It represents a phase shift of radians.

Example 3: Graph a function by converting it to or

Graph by converting it to .

From the worked example in the unit,

The graph is a cosine function with an amplitude of 2, period of and the phase shifted to the right, radians.



Lesson: Solving equations of the form

Example 1: Solve in the domain

Let

Expand the right hand side:

Equate the coefficients: and 3

Since sine and cosine are positive, must be a 1st quadrant angle.

Find by forming a right angle triangle in the 1st quadrant and applying Pythagoras’ theorem or using the trigonometric identity Obtain

Find

Solving in the domain is equivalent to solving in the domain

 or

Note: which can be observed by forming a right angle triangle.

The solution can be checked by graphing and and reading the points of intersection or by graphing and reading the intercepts in the domain. The graphical solutions shown use dotted blue lines to signify the domain.



Lesson: Solving equations using factorisation and/or compound angle results

Example 1: Solve in the domain

Since the tangent ratio is positive and is in the 1st and 3rd quadrants.

 or , check both solutions line in the domain

 or

Check the solutions by considering the points of intersection of and



Example 2: Solve in the domain

Since the sine ratio is positive and is in the 1st and 2nd quadrants

 or , check both solutions line in the domain

 or

Check the solutions by considering the points of intersection of and .



Example 3: Solve in the domain

Since the sine ratio is positive and is in the 1st and 2nd quadrants

 or , check both solutions line in the domain

 lies outside this domain. Convert this to an equivalent angle in the domain i.e.



 or

 or

Check the solutions by considering the points of intersection of and .



Without adjusting for the domain of , we would have obtained the solutions and . The second of which can be observed to lie outside the given domain.

Example 4: Solve in the domain

Substitute

 or

 or

The solution can be checked by considering the intercepts of the graph of or .



Lesson: Solving equations using double angle results

Example 1: Solve in the domain

The tangent ratio is positive so the solution will lie in the 1st and 3rd quadrants.

Check the solutions by considering the points of intersection of and

 

Example 2: Solve in the domain

Substitute

The cosine ratio is positive so the solution will lie in the 1st and 4th quadrants.

Check the solutions by considering the points of intersection of and



Example 3: Solve in the domain

Substitute

The first sine ratio is positive so the solution will lie in the 1st and 2nd quadrants

Check solutions from the intercepts of the graph of



Example 4: Solve in the domain

Substitute

The cosine ratio is positive, so the solution to this ratio will lie in the 1st and 4th quadrants

Check solutions from the intercepts of the graph of



Lesson: Solving equations using the t-formulae

Example 1: Solve using the result
 in the domain

Given then substitute and

 or

The 1st tangent ratio is negative and will lie in the 2nd quadrant. The 2nd tangent ratio is positive and will lie in the first quadrant.

Lesson: Proofs and applications of trigonometric identities

Example 1:

Example 2:

Example 3:

Example 4:

Example 5: Prove . Hence or otherwise solve in the domain

Prove

Now, and

The sine ratio is positive so the solution will lie in the 1st and 2nd quadrants.

Check the solutions by considering the points of intersection of and

