 Year 11 mathematics extension 1

| ME-T2 Further Trigonometric Identities | Unit duration |
| --- | --- |
| The topic Trigonometric Functions involves the study of periodic functions in geometric, algebraic, numerical and graphical representations. It extends to exploration and understanding of inverse trigonometric functions over restricted domains and their behaviour in both algebraic and graphical form.A knowledge of trigonometric functions enables the solving of problems involving inverse trigonometric functions, and the modelling of the behaviour of naturally occurring periodic phenomena such as waves and signals to solve problems and to predict future outcomes.The study of the graphs of trigonometric functions is important in developing students’ understanding of the connections between algebraic and graphical representations and how this can be applied to solve problems from theoretical or real-life scenarios and situations. | 4 weeks |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is for students to define and work with trigonometric identities to both prove results and manipulate expressions.Students develop knowledge of how to manipulate trigonometric expressions to solve equations and to prove results. Trigonometric expressions and equations provide a powerful tool for modelling quantities that vary in a cyclical way such as tides, seasons, demand for resources, and alternating current. The solution of trigonometric equations may require the use of trigonometric identities. | A student:* uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
* applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems ME11-3
* uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
* communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7
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| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| The material in this topic builds on content from the Mathematics advanced topic of MA-F1 Working with functions and the Mathematics extension 1 topics of ME-F1 Further work with functions and ME-T1 Inverse trigonometric functions. | * Summative Assessment: Investigating Trigonometric Functions. This investigative assignment involves the graphing and exploring of Inverse Trigonometric functions and Further Trigonometric Identities. (assessment of learning)
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All outcomes referred to in this unit come from [Mathematics Extension 1](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) Syllabus
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Glossary of terms

| Term | Description |
| --- | --- |
| even function **** | Algebraically, a function is even if , for all values of in the domain.An even function has line symmetry about the y-axis. |
| identity  | An identity is a statement involving a variable(s) that is true for all possible values of the variable(s). |
| odd function  | Algebraically, a function is odd if , for all values of in the domain.An odd function has point symmetry about the origin. |
| range (of function)  | The range of a function is the set of values of the dependent variable for which the function is defined. |

| Lesson sequence | ContentStudents learn to: | Suggested teaching strategies and resources  | Date and initial | Comments, feedback, additional resources used |
| --- | --- | --- | --- | --- |
| Introducing sum and difference of angles results(2 - 3 lessons) | * derive and use the sum and difference expansions for the trigonometric functions , and (ACMSM044)
 | **Assumed knowledge*** Students need to be familiar with sine and cosine rules.

**Deriving the sum and difference of angles trigonometric identity for cosine*** Start by defining a point P on a unit circle such that and and . Similarly, define . Find the distance PQ using two methods

Method 1: Use the cosine ruleMethod 2: Use the distance between two pointsEquating the results gives the sum of angles identity* Substituting with into the sum of angles identity and using the odd and even properties for sine and cosine gives the difference of angles identity for cosine

**Deriving the sum and difference of angles trigonometric identity for sine*** Substitute into the difference of angles identity for cosine derived above.

 Matching LHS and RHS gives the sum of angles identity for sine* Substituting with into the sum of angles identity for sine, derived above, and using the odd and even properties for sine and cosine gives the difference of angles identity for sine

 **Deriving the sum and difference of angles trigonometric identity for tan*** Define and using sum of angles identities derived above gives
* Dividing the numerator and denominator by gives the sum of angles identity for tan

 * Similarly, by using the difference of angles identities gives the difference of angle identity for tan

**Applying the trigonometric identities*** Students need to be exposed to, but not limited to, examples in the form of
	+ Show
	+ Simplify finding the answer in exact form.
	+ Find the exact value of
* Students develop knowledge of how to manipulate trigonometric expressions to solve equations and to prove results.
 |  |  |
| Applying double angles results(2 lessons) | * derive and use the double angle formulae for , and (ACMSM044) Literacy icon
*
*
*
 | **Deriving double angle formulae*** Using sum of angle identities for sine, cosine and tan, from above, and substituting gives
	+ Note, there are three double angle results for cosine, as follows, that students need to be familiar with.

 (as ) (as ) |  |  |
| Using t-formulae(2 lessons) | * derive and use expressions for , and in terms of where (the -formulae)
 | **Deriving results for sine, cosine and tan using t-formulae*** Start by defining
* Draw a right-angled triangle, labelling the opposite side as and the adjacent side equal to , to illustrate the above property.

A right angle triangle with the opposite side labelled as t and the adjacent side equal to one.* Use Pythagoras’ theorem to gives the hypotenuse equal to and therefore the right-angled triangle above becomes

The right-angled triangle from earlier with the opposite side labelled as t, the adjacent side equal to one and the hypotenuse equal to the square root of 1 plus t squared.* Interpreting this triangle gives the results

and * Using these results when applying the double angle results gives

 **Using t-formulae to simplify trigonometric expressions or solve equations*** Staff can use [Trigonometry: Lesson 4 (identities: t formulas)](https://www.youtube.com/watch?v=bDV5I6jhVx0) (duration 8:38)to manipulate trigonometric expressions
* Staff can use this clip [Using t-results to Solve Trigonometric Equations (Example 1)](https://www.youtube.com/watch?v=oh9psFquVoo) (duration 12:04) to solve trigonometric equations.
 |  |  |
| Applying product as sums and differences results(2 lessons) | * derive and use the formulae for trigonometric products as sums and differences for , , and (ACMSM047) Literacy icon
 | **Verifying products as sums results*** Staff should demonstrate or lead students towards, at least, one of the products as sums and differences results.

For example, start by stating the sum and difference results for cosineAsk students to consider what would happen if the equations were added together? Which term(s) would be eliminated?Adding the left and right hand sides of the equations givesRearranging gives the product as sum result**Applying the product as sums and differences results*** Students need to be familiarised with using the product as sums and differences results in reverse, especially as a technique for solving equations.
* Staff can access these following resources:
* A [tutorial by dadeschools.net](http://teachers.dadeschools.net/lberkson/Documents/Ch5_Section4.pdf) showing product as sum and sum as product results
* As similar [tutorial style resource from asurams.edu](https://gateway.asurams.edu/affordable-learning-program/14-SumToProductAndProductToSumFormulas/14-Sum2ProductAndProduct2Sum.pdf).
* Students should be introduced, but limited to, examples of the form
* Find the exact value of
* By using the sum as product result, solve the equation
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Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in Comments, Feedback, Additional Resources Used sections.