 Year 11 mathematics extension 1

| ME-F1.1 Graphical relationships | Unit duration |
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| The topic Functions involves the use of both algebraic and graphical conventions and terminology to describe, interpret and model relationships of and between changing quantities. This topic provides the means to more fully understand the behaviour of functions, extending to include inequalities, absolute values and inverse functions. A knowledge of functions enables students to discover connections between algebraic and graphical representations, to determine solutions of equations and to model theoretical or real-life situations involving algebra. The study of functions is important in developing students’ ability to find, recognise and use connections, to communicate concisely and precisely, to use algebraic techniques and manipulations to describe and solve problems, and to predict future outcomes in areas such as finance, economics and weather. | 1.5 weeks |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is to further explore functions in a variety of contexts including: reciprocal and inverse functions, manipulating graphs of functions, and parametric representation of functions. The study of inequalities is an application of functions and enables students to express domains and ranges as inequalities. Students develop proficiency in methods to identify solutions to equations both algebraically and graphically. The study of inverse functions is important in higher Mathematics and the calculus of these is studied later in the course. The study of parameters sets foundations for later work on projectiles. | A student:* uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
* manipulates algebraic expressions and graphical functions to solve problems ME11-2
* uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
* communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7
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| Prerequisite knowledge | Assessment strategies |
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| Students should have studied the concepts explored in the year 11 Mathematics advanced topic of MA-F1 Working with functions. | * In pairs, students are to write a function for their partner to sketch, then they must check and give feedback for the graph sketched. The graphs can include composite functions. This activity may be more engaging with mini-whiteboards or butcher’s paper and markers.
* Learning journal
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All outcomes referred to in this unit come from [Mathematics Extension 1](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) Syllabus
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Glossary of terms

| Term | Description |
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| composite function | In a composite function, the output of one function becomes the input of a second function. More formally, the composite of 𝑓 and 𝑔, acting on 𝑥, can be written as ((𝑥)), with (𝑥) being performed first.  |
| reciprocal | The reciprocal of a quantity $x$ is define formally as $\frac{1}{x}$. The product of a quantity and its reciprocal is equal to 1.Informally, a reciprocal of a quantity, expressed as a fraction, can be generated by reversing its numerator and denominator, eg) the reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$. |

Lesson sequence

| Lesson Sequence | ContentStudents learn to: | Suggested teaching strategies and resources  | Date and initial | Comments, feedback, additional resources used |
| --- | --- | --- | --- | --- |
| Reciprocal functions(1 lesson) | * examine the relationship between the graph of $y=f\left(x\right)$ and the graph of $y=\frac{1}{f(x)}$ and hence sketch the graphs (ACMSM099)
 | **Introduction** * Teachers should consider introducing the topic by outlining all the different manipulations of $y=f\left(x\right)$ that will be explored. This introduction should include examining the various critical points or regions of different functions, including considerations for where the graph will be negative, positive or undefined but also what happens at $x=0$, $x=\pm 1$ and when $-1<x<1$. The emphasis here should be that students do not necessarily need to know the equation of the function, but they should be able to identify areas where the resulting function will be equal to, greater than or less than $y=f\left(x\right)$.

**Reciprocal functions*** Students need to consider that dividing 1 by a number greater than 1 results in a value between 0 and 1. Also, dividing 1 by a number between 0 and 1 results in a number greater than 1.
* The graph will be undefined or discontinuous when $f(x)=0$, where there is no reciprocal function
* Use the [desmos](http://www.desmos.com/calculator) template [Reciprocal Functions](https://www.desmos.com/calculator/orpozkxgty) to show patterns of change when taking the reciprocal of a function. This template can be modified to explore the reciprocal of additional functions.
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| Squares and square root functions(1 lesson) | * examine the relationship between the graph of $y=f\left(x\right)$ and the graphs of $y^{2}=f(x)$ and $y= \sqrt{f(x)}$ and hence sketch the graphs
 | **Squares and square root functions*** Students to consider the squares and square roots of a range of functions studied within the scope of the syllabus, including and not limited to polynomials, hyperbolic functions, exponential functions, logarithmic functions and trigonometric functions.
* Students need to consider that taking the square-root of a number greater than 1 results in a smaller value. Also, taking the square-root of a number between 0 and 1 results in a number greater than 1.
* The graph will have no real solution when $f(x)<0$,
* Use the [desmos](http://www.desmos.com/calculator) template [Square Root Functions](https://www.desmos.com/calculator/q0aab3ley6) to show patterns of change when taking the square root of a function. This template can be modified to explore the square roots of additional functions. It also compares the functions to those of the form $y^{2}=f(x)$
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| Absolute value functions(1 lesson) | * examine the relationship between the graph of $y=f\left(x\right)$ and the graphs of $y=|f(x)| $and $y=f(|x|)$ and hence sketch the graphs (ACMSM099)
 | **Absolute value functions*** Consider the graphs of $y=|f(x)| $and $y=f(|x|)$ of functions studied within the scope of the syllabus, including but not limited to polynomials, hyperbolic functions, exponential functions, logarithmic functions and trigonometric functions.
* Use the [desmos](http://www.desmos.com/calculator) templates [Absolute Value Functions (1)](https://www.desmos.com/calculator/oyto4wsswg) and [Absolute Value Functions (2)](https://www.desmos.com/calculator/mawnmltpak) to show patterns of change when exploring the graphs of $y=|f(x)| $and $y=f(|x|)$. This template can be modified to explore the graphs of various functions of the form $y=|f(x)| $and $y=f(|x|)$.
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| Sum and product of functions(2 lessons) | * examine the relationship between the graphs of $y=f(x)$ and $y=g(x)$ and the graphs of $y=f(x)+g(x)$ and $y=f(x)g(x)$ and hence sketch the graphs
* apply knowledge of graphical relationships to solve problems in practical and abstract contexts **AAM**
 | **Sum and product of functions*** Use considerations of the domain and range of composite functions when graphing.
* Introduce and explore composite functions that involve adding or multiplying polynomials, hyperbolic functions, exponential functions, logarithmic functions, trigonometric functions with the aid of a graphing calculator such as [desmos](https://www.desmos.com/calculator).
* Use the [desmos](http://www.desmos.com/calculator) templates [Addition of Ordinates](https://www.desmos.com/calculator/3g6z9h0eo8) and [Multiplication of Ordinates](https://www.desmos.com/calculator/ju1ojp13qo) to show patterns of change when exploring the graphs of the form $y=f(x)+g(x)$ and $y=f(x)g(x)$.
* Draw $y=f\left(x\right) $and $y=g\left(x\right)$ first on the same plane and then attempt to draw $y=f(x)+g(x)$ and $y=f(x)g(x)$ over the top by considering the sum or product of y-values at key points along the graph
* Students to complete their own sum and product of functions with simple graphs. **Resource:** sum-and-product-of-functions.DOCX
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Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.