 Year 12 Mathematics Extension 1

| ME-C2 Further calculus skills | Unit duration |
| --- | --- |
| The topic Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It involves the development of analytic and numeric integration techniques and the use of these techniques in solving problems.  The study of calculus is important in developing students’ knowledge, understanding and capacity to operate with and model situations involving change, and to use algebraic and graphical techniques to describe and solve problems and to predict future outcomes with relevance to, for example science, engineering, finance, economics and the construction industry. | 6 lessons |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is to further develop students’ knowledge, skills and understanding relating to differentiation and integration techniques.  Students develop an awareness and understanding of the interconnectedness of topics across the syllabus, and the fluency that can be obtained in the use of calculus techniques. Later studies in mathematics place prime importance on familiarity and confidence in a variety of calculus techniques as these are used in many different fields. | A student:   * applies techniques involving proof or calculus to model and solve problems ME12-1 * uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution ME12-4 * chooses and uses appropriate technology to solve problems in a range of contexts ME12-6 * evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7 |

| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| Students should have studied MA-T2 Trigonometric functions and identities, MA-T3 Trigonometric functions and graphs, MA-C2 Differential calculus, ME-T1 Inverse Trigonometric functions and ME-T2 Further Trigonometric Identities. | * Formative assessment: This unit allows students to develop their fluency and understanding of calculus. Staff should use activities that allow students to build mastery through formative assessment techniques such as pre and post testing, mini whiteboard activities and exit slips |

All outcomes referred to in this unit come from the [Mathematics Extension 1](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) syllabus  
© NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2017

Glossary of terms

| Term | Description |
| --- | --- |
| integrand | An integrand is a function that is to be integrated. |
| substitution | Substitution is a technique that defines and uses a parameter to convert an expression or equation, without changing the integrity of the expression or equation. |

| **Lesson sequence** | **Content** | **Suggested teaching strategies and resources** | **Date and initial** | **Comments, feedback, additional resources used** |
| --- | --- | --- | --- | --- |
| Integrating using substitution  (1 or 2 lessons) | * find and evaluate indefinite and definite integrals using the method of integration by substitution, using a given substitution Critical and creative thinking icon | **Note:** For solutions to the exemplar questions from the NESA topic guidance, see me-c2-nesa-exemplar-question-solutions.DOCX. This contains questions and solutions from throughout the topic.  **Integrating using substitution**   * **Note:** In Mathematics Extension 1, students will need to use this technique with a given expression to substitute. In Mathematics Extension 2, students may need to identify and define the expression to substitute. * The technique of substitution uses the property   where   * The expression for substitution can be identified by recognising and in the integrand, then defining . Note that the parameter is generally used but this is not mandatory. * The technique of substitution is used to make complicated integrals simpler to solve. * Students need to be exposed to examples of the type:   + given   + given   + given   **Solutions:** integrating-by-substitution.DOCX |  |  |
| Integrating powers of trigonometric functions  (1 or 2 lessons) | * prove and use the identities and to solve problems * solve problems involving and Critical and creative thinking icon | **Integrating powers of trigonometric functions**   * Students need to be shown the proof of one of the identities for or , and produce the proof for the other using the techniques demonstrated.   Let and  from , substitute  Let  as shown   * Build on understanding of the reverse chain rule, from the subtopic MA-C4 The anti-derivative. In particular, the reverse chain rule techniques can only be applied to linear expressions of within a function, **i.e.** where . Therefore the integrals and cannot be integrated in this form because is not linear, **i.e.** equals or * Apply the appropriate identity for or prior to integrating.   Let |  |  |
| Using calculus with trigonometric functions  (1 lesson) | * find derivatives of inverse functions by using the relationship * solve problems involving the derivatives of inverse trigonometric functions * integrate expressions of the form or (ACMSM121) | **Differentiating inverse trigonometric functions**   * Examine the techniques to prove the derivative results for , and .   **Resource:** differentiating-inverse-trig-functions.DOCX  **Integrating inverse trigonometric functions**   * Building on the understanding of the derivatives for inverse trigonometric functions, students need to establish and use the integral results for and |  |  |
| Change an integral into a standard form  (1 lesson) | * find and evaluate indefinite and definite integrals using the method of integration by substitution, using a given substitution Critical and creative thinking icon   + change an integrand into an appropriate form using algebra | **Change an integral into a standard form**   * The key learning intention is for students to use a mixture of algebraic manipulation skills and substitution techniques to change integrals into a standard form found on the NESA HSC reference sheet on the [NESA Mathematics Extension 1 page](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017).   **Resource:** changing-the-integral-activty.DOCX |  |  |

Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.