 Sample proportions and binomial probability

These questions come from the NESA sample unit for The Binomial Distribution for the [Mathematics Extension 1](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) syllabus   
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Part 1 – questions

1. Suppose that 45% of all HSC students exercise at least 4 days each week. If a random sample of 50 students is taken, what is the probability that at least 80% of them exercise at least 4 days per week?
2. It is known that 24% of HSC students do not have a driver’s licence. In a random sample of 16 HSC students, what is the probability that half of them will not have a driver’s licence?
3. A computer simulation is designed to draw random samples of size from a large dataset. The proportion of the population that exhibits a certain characteristic is . If represents the sample proportion exhibiting the characteristic under investigation, find the largest sample size that should be used so that the standard deviation of is at least 0.01.
4. A manufacturer makes earbuds that have a probability of 0.02 of being defective. Quality control officers test random samples of 50 earbuds each hour and reject the earbuds made in that hour if at least 3 earbuds are defective. Find the probability that the earbuds made in any hour will be rejected. Answer to 2 significant figures.
5. It is estimated that approximately 45% of Australians will experience a mental health condition in their lifetime. If a random sample of 120 mature adults were surveyed, what is the probability of 48 or more having experienced a mental health condition? (Refer: [Beyond Blue](https://www.beyondblue.org.au/the-facts))

Part 1 – worked solutions

Note: For all questions, apply the tests and to determine if the distribution of the sample proportions can be approximated using the normal distribution.

1. Suppose that 45% of all HSC students exercise at least 4 days each week. If a random sample of 50 students is taken, what is the probability that at least 80% of them exercise at least 4 days per week?

The distribution of the sample proportions can be approximated using the normal distribution with:

The probability that at least 80% of them exercise at least 4 days per week is approximately zero.

Alternatively solve using binomial probability,

1. It is known that 24% of HSC students do not have a driver’s licence. In a random sample of 16 HSC students, what is the probability that half of them will not have a driver’s licence?

The distribution of the sample proportions **cannot** be approximated using the normal distribution.

Solve using binomial probability, 0.016

1. A computer simulation is designed to draw random samples of size from a large dataset. The proportion of the population that exhibits a certain characteristic is . If represents the sample proportion exhibiting the characteristic under investigation, find the largest sample size that should be used so that the standard deviation of is at least 0.01.

Assuming the distribution of the sample proportions can be approximated using the normal distribution then:

Solve for n such that

The largest sample size that can be used is 1875.

1. A manufacturer makes earbuds that have a probability of 0.02 of being defective. Quality control officers test random samples of 50 earbuds each hour and reject the earbuds made in that hour if at least 3 earbuds are defective. Find the probability that the earbuds made in any hour will be rejected. Answer to 2 significant figures.

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The distribution of the sample proportions **cannot** be approximated using the normal distribution.

Solve using binomial probability:

1. It is estimated that approximately 45% of Australians will experience a mental health condition in their lifetime. If a random sample of 120 mature adults were surveyed, what is the probability of 48 or more having experienced a mental health condition? (Reference: [Beyond Blue](https://www.beyondblue.org.au/the-facts))

The distribution of the sample proportions can be approximated using the normal distribution with:

48 represents a sample proportion of