 Sample proportions and binomial probability

These questions come from the NESA sample unit for The Binomial Distribution for the [Mathematics Extension 1](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) syllabus
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Part 1 – questions

1. Suppose that 45% of all HSC students exercise at least 4 days each week. If a random sample of 50 students is taken, what is the probability that at least 80% of them exercise at least 4 days per week?
2. It is known that 24% of HSC students do not have a driver’s licence. In a random sample of 16 HSC students, what is the probability that half of them will not have a driver’s licence?
3. A computer simulation is designed to draw random samples of size $n$ from a large dataset. The proportion of the population that exhibits a certain characteristic is $p=0.25$. If $\hat{p}$ represents the sample proportion exhibiting the characteristic under investigation, find the largest sample size that should be used so that the standard deviation of $\hat{p}$ is at least 0.01.
4. A manufacturer makes earbuds that have a probability of 0.02 of being defective. Quality control officers test random samples of 50 earbuds each hour and reject the earbuds made in that hour if at least 3 earbuds are defective. Find the probability that the earbuds made in any hour will be rejected. Answer to 2 significant figures.
5. It is estimated that approximately 45% of Australians will experience a mental health condition in their lifetime. If a random sample of 120 mature adults were surveyed, what is the probability of 48 or more having experienced a mental health condition? (Refer: [Beyond Blue](https://www.beyondblue.org.au/the-facts))

Part 1 – worked solutions

Note: For all questions, apply the tests $np\geq 10$ and $n\left(1-p\right)\geq 10$ to determine if the distribution of the sample proportions can be approximated using the normal distribution.

1. Suppose that 45% of all HSC students exercise at least 4 days each week. If a random sample of 50 students is taken, what is the probability that at least 80% of them exercise at least 4 days per week?

$$n=50, p=0.45$$

$$np=22.5$$

$$n\left(1-p\right)=27.5$$

The distribution of the sample proportions can be approximated using the normal distribution with:

$$μ\_{\hat{p}}=p=0.45$$

$$σ\_{\hat{p}}=\sqrt{\frac{p(1-p)}{n}}=\sqrt{\frac{0.45(1-0.45)}{50}}=0.070356…≈0.07$$

$$z\_{0.8} =\frac{0.8-0.45}{0.07}=5$$

The probability that at least 80% of them exercise at least 4 days per week is approximately zero.

Alternatively solve using binomial probability,
$$P\left(X\geq 40\right)=P\left(X=40\right)+P\left(X=41\right)+…+P(X=50)$$

1. It is known that 24% of HSC students do not have a driver’s licence. In a random sample of 16 HSC students, what is the probability that half of them will not have a driver’s licence?

$$n=16, p=0.24$$

$$np=3.84$$

$$n\left(1-p\right)=12.16$$

The distribution of the sample proportions **cannot** be approximated using the normal distribution.

Solve using binomial probability, $P\left(X=8\right)= ^{16}C\_{8}×0.24^{8}×0.76^{8}≈$0.016

1. A computer simulation is designed to draw random samples of size $n$ from a large dataset. The proportion of the population that exhibits a certain characteristic is $p=0.25$. If $\hat{p}$ represents the sample proportion exhibiting the characteristic under investigation, find the largest sample size that should be used so that the standard deviation of $\hat{p}$ is at least 0.01.

$$n=?, p=0.25$$

Assuming the distribution of the sample proportions can be approximated using the normal distribution then:

Solve for n such that $σ\_{\hat{p}}\geq 0.01$

$$σ\_{\hat{p}}=\sqrt{\frac{p(1-p)}{n}}=\sqrt{\frac{0.25(1-0.25)}{n}}\geq 0.01$$

$$\sqrt{\frac{0.1875}{n}}\geq 0.01$$

$$\frac{0.1875}{n}\geq 0.0001$$

$$0.1875\geq 0.0001n$$

$$n\leq 1875$$

The largest sample size that can be used is 1875.

1. A manufacturer makes earbuds that have a probability of 0.02 of being defective. Quality control officers test random samples of 50 earbuds each hour and reject the earbuds made in that hour if at least 3 earbuds are defective. Find the probability that the earbuds made in any hour will be rejected. Answer to 2 significant figures.

$$n=50, p=0.02$$

$np=$1

$$n\left(1-p\right)=49$$

The distribution of the sample proportions **cannot** be approximated using the normal distribution.

Solve using binomial probability:

$$P\left(X\geq 3\right)=1-P\left(X<3\right)=1-P\left(X=0\right)-P\left(X=1\right)-P(X=2)$$

$$P\left(X\geq 3\right)=1- ^{50}C\_{0}×0.02^{0}×0.98^{50}- ^{50}C\_{1}×0.02^{1}×0.98^{49}- ^{50}C\_{2}×0.02^{2}×0.98^{48}$$

$$P\left(X\geq 3\right)=1-0.9215…$$

$P\left(X\geq 3\right)≈0.078$

1. It is estimated that approximately 45% of Australians will experience a mental health condition in their lifetime. If a random sample of 120 mature adults were surveyed, what is the probability of 48 or more having experienced a mental health condition? (Reference: [Beyond Blue](https://www.beyondblue.org.au/the-facts))

$$n=120, p=0.45$$

$$np=54$$

$$n\left(1-p\right)=66$$

The distribution of the sample proportions can be approximated using the normal distribution with:

$$μ\_{\hat{p}}=p=0.45$$

$$σ\_{\hat{p}}=\sqrt{\frac{p(1-p)}{n}}=\sqrt{\frac{0.45(1-0.45)}{120}}=0.05$$

48 represents a sample proportion of $0.4$

$z score of 0.4 =\frac{0.45-0.4}{0.05}=-1$

$$P(X\geq 50)≈0.84$$