 Question bank

Question bank for proof by induction

Part A – summative proofs

Prove by mathematical induction that the following formulas are true for every positive integer value of n.

| Number | Equation |
| --- | --- |
| 1 | $$1+2+3+4+ ……………………….n= \frac{n(n+1)}{2}$$ |
| 2 | $$\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+ ………………………………………+\frac{n}{2}= \frac{n(n+1)}{4}$$ |
| 3 | $$2+4+6+ …..+ 2n=n(n+1)$$ |
| 4 | $$3+6+9+ …………………………….+ 3n= \frac{3n(n+1)}{2}$$ |
| 5 | $$4+8+12+ …………………………….+ 4n=2n(n+1)$$ |
| 6 | $$2+6+10+ ……….+\left(4n-2\right)=2n^{2}$$ |
| 7 | $$1+5+9+ ………..+\left(4n-3\right)=n(2n-1)$$ |
| 8 | $$1+3+5+ ……………………+\left(2n-1\right)= n^{2}$$ |
| 9 | $$2+5+8+ ……………………..+\left(3n-1\right)= \frac{n }{2}(3n+1)$$ |
| 10 | $$1^{3}+2^{3}+3^{3}+ …………………….+ n^{3}= \frac{n^{2}}{4}(n+1)^{2}$$ |

Part B – divisibility proofs

Prove by mathematical induction that the following formulas are true for every positive integer value of n.

| Number | Equation |
| --- | --- |
| 1 |  $n\left(n+1\right)\left(n+2\right)$ is divisible by 3. |
| 2 | $3^{3n}+2^{n+2}$ is divisible by 5. |
| 3 | The sum of the cubes of three consecutive integers is divisible by 3. |
| 4 | $n\left(n+1\right)$ is an even number **i.e.** divisible by 2 |
| 5 | $7^{n}-1$ is divisible by 3. |
| 6 | $5^{n}+3$ is divisible by 4. |
| 7 | $4^{n}-1$ is divisible by 3 for all $n\geq 1$. |
| 8 |  $n^{2}+n$ is divisible by 2 for all positive integers $n$. |
| 9 | If $x\ne a, $then $x^{n}-a^{n}$ is divisible by $x-a$ for all positive integers $n$. (**Hint:** Add and subtract $xa^{k}$ to $x^{k+1}-a^{k+1}$ then group terms and factor.) |
| 10 | If $x\ne y, $then $x^{2n}-y^{2n}$ is divisible by $x-y$ for all positive integers $n. $ |