 Number of handshakes

Part 1 – investigation

1. At a conference, every person shakes hands with every other person at the conference. If there are three people at the conference, Andrew, Bill and Conrad, there are 3 handshakes:
	* Andrew and Bill shake hands
	* Andrew and Conrad shake hands, and
	* Bill and Conrad shake hands

Complete the table below, indicating the total number of handshakes for conferences with between 1 and 8 people in attendance.

| Number of people | Minimum number of handshakes |
| --- | --- |
| 1 |  |
| 2 |  |
| 3 | 3 |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

1. Create a spreadsheet to calculate the total number of handshakes for conferences of sizes 1 person up to 50 people. Write in the space below the number of handshakes at a 50 person conference.

1. How would you calculate the number of handshakes at a conference with 51 people?

1. If you know that there are $y$ handshakes at a conference of $n-1$ people, what would be the number of handshakes at a conference of $n$ people?

Part 2 – proof of formula

There are two ways to find the total number of handshakes at a conference.

Series

We know that if 3 people are at the conference, there will be 3 handshakes. So if we let p be the number of people and n be the number of handshakes, when$ p=3, n=3$.

When $p=4,$ a fourth person arrives and shakes hands with all existing members of the group, a total of 3. Hence the handshake total becomes $n = 3 + 3 = 6. $

When $p = 5, $a fifth person arrives, shaking hands with the existing 4 members, hence the total number of handshakes becomes $n = 6 + 4 = 10. $

When $p = 1, n = 0$(one person has no one to shake hands with), therefore our series from the beginning becomes:

$$n = 0 + 1 + 2 + 3 + 4 + … + (p – 1)$$

Formula

For a given number of people, there are p terms in the series. If we examine the series when p = 5, we find that the total number of handshakes is

$$n = 0 + 1 + 2 + 3 + 4 = 10. $$

If we average these 5 terms, we get$ \frac{ 0+1+2+3+4}{5}=2$.

Therefore $n = 2 + 2 + 2 + 2 + 2 = 10.$

The series will always be symmetrical, and hence we can treat each term as the midpoint of the greatest and smallest term.

For example – we can treat each term of the series as if they were $\frac{p-1}{2}. $Since there are p terms, the formula for the number of handshakes at a conference of p people is $n=\frac{p(p-1)}{2}$.

Proof

Clearly the formula is far more useful than the series.

Prove by mathematical induction that

$n=0+1+2+3+4+…+\left(p-1\right)=\frac{p(p-1)}{2}$ for integral $p$ with $p$ with $p\geq 1$.