 Number of handshakes

Part 1 – investigation

1. At a conference, every person shakes hands with every other person at the conference. If there are three people at the conference, Andrew, Bill and Conrad, there are 3 handshakes:
   * Andrew and Bill shake hands
   * Andrew and Conrad shake hands, and
   * Bill and Conrad shake hands

Complete the table below, indicating the total number of handshakes for conferences with between 1 and 8 people in attendance.

| Number of people | Minimum number of handshakes |
| --- | --- |
| 1 |  |
| 2 |  |
| 3 | 3 |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

1. Create a spreadsheet to calculate the total number of handshakes for conferences of sizes 1 person up to 50 people. Write in the space below the number of handshakes at a 50 person conference.

1. How would you calculate the number of handshakes at a conference with 51 people?

1. If you know that there are handshakes at a conference of people, what would be the number of handshakes at a conference of people?

Part 2 – proof of formula

There are two ways to find the total number of handshakes at a conference.

Series

We know that if 3 people are at the conference, there will be 3 handshakes. So if we let p be the number of people and n be the number of handshakes, when.

When a fourth person arrives and shakes hands with all existing members of the group, a total of 3. Hence the handshake total becomes

When a fifth person arrives, shaking hands with the existing 4 members, hence the total number of handshakes becomes

When (one person has no one to shake hands with), therefore our series from the beginning becomes:

Formula

For a given number of people, there are p terms in the series. If we examine the series when p = 5, we find that the total number of handshakes is

If we average these 5 terms, we get.

Therefore

The series will always be symmetrical, and hence we can treat each term as the midpoint of the greatest and smallest term.

For example – we can treat each term of the series as if they were Since there are p terms, the formula for the number of handshakes at a conference of p people is .

Proof

Clearly the formula is far more useful than the series.

Prove by mathematical induction that

for integral with with .