 Intersecting lines

Part 1 – Investigation

Consider the number of enclosed regions formed by intersecting lines, with the rules that:

* There are no parallel lines
* There are no sets of three or more concurrent lines

In Figure 1, we can see 2 intersecting lines which clearly have no enclosed regions.

In Figure 2, we can see 3 intersecting lines which clearly have one enclosed region (shaded).





Investigate this pattern by constructing similar diagrams for n intersecting lines and complete the table below.

| Intersecting lines | Number of enclosed regions |
| --- | --- |
| 2 | 0 |
| 3 | 1 |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

How would you calculate the number enclosed regions formed by 10 intersecting lines.

How would you calculate the number enclosed regions formed by n intersecting lines?

Part 2 – Proof of formula

There are two ways to find the number of enclosed regions formed by n intersecting lines.

Series – If we choose to draw the additional line outside the existing intersections as in Figure 3, we can see that when we add the 4th line, it intersects 3 existing lines, forming 2 additional regions.

This gives rise to the idea that three lines form 1 regions,

Four lines form 1 + 2 regions,

Five lines form 1 + 2 + 3 regions and so forth.

Hence the series that gives the numbers of enclosed regions formed by n intersecting lines is

1 + 2 + 3 + …. + (n – 2)



Formula – The formula $\frac{(n-2)(n-1)}{2}$ will also give the total number of enclosed regions formed by *n* intersecting lines, and is a much simpler way of calculating solutions for large numbers of lines.

**Proof –** Prove by Mathematical Induction that

1 + 2 + 3 + … + (*n* – 2) = $\frac{(n-2)(n-1)}{2}$ for integral values of *n* with *n* ≥ 2