 Diagonals of polygons

Part 1 – investigation

Consider the number of diagonals that exist within an n sided convex polygon.

We know that a triangle has no diagonals, and in Figure 1 we can see that a quadrilateral has 2 diagonals. Complete the table below to identify the number of diagonals in each n sided polygon



| n – Number of sides | Number of diagonals |
| --- | --- |
| 3 | 0 |
| 4 | 2 |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |

How would you calculate the number diagonals existing in a polygon of 10 side?

How would you calculate the number diagonals existing in a polygon of n sides?

Part 2 – proof of formula

There are two ways to find the number of diagonals existing in a polygon n sides.

Series – Consider the tree shapes below:



The number of diagonals for these three figures are 2, 5 and 9.

To get from one to the next, we add the next natural number. So a three sided polygon has 0 diagonals,

* A four sided polygon has 0 + 2 = 2 diagonals
* A five sided polygon has 0 + 2 + 3 = 5 diagonals
* A six sided polygon has 0 + 2 + 3 + 4 = 9 diagonals and continuing this we would see that
* A seven sided polygon will have 0 + 2 + 3 + 4 + 5 = 14 diagonals.

Hence an n sided polygon will have 2 + 3 + 4 + 5 + … + (n – 2) diagonals.

Formula – Notice that Figure 1 has 4 vertices, and each vertex connects to one diagonal. This would suggest 4 diagonals, but as each diagonal has a beginning and an end, we need to divide by 2 to get 2 diagonals.

In Figure 2, we have 5 vertices with each vertex connecting to two diagonals. This would be 5 x 2 = 10 diagonals, but similarly we divide by 2 to get 5 diagonals.

In Figure 3, we have 6 vertices, each connecting to 3 diagonals, so 6 x 3 ÷ 2 = 9 diagonals.

A seven sided figure would have 7 vertices with each connecting to 4 diagonals, so 7 x 4 ÷ 2 = 14 diagonals.

Hence a formula should be $\frac{n(n-3)}{2}$.

Proof – Prove by Mathematical Induction that

2 + 3 + 4 + 5 + … + (n – 2) = $\frac{n(n-3)}{2}.$