 NESA exemplar question solutions

Integrating using substitution

Examples of integrals to be determined using given substitutions include:

1. Use the substitution to determine

Substitution:

Let

1. Use the substitution to evaluate

Substitution:

Bounds:

Let

1. Use the substitution to determine

Substitution:

Let

Integrating powers of trigonometric functions

Examples of integrals to be determined involving squares of the trigonometric functions sine and cosine, and those that can be found by a simple substitution, include:

Use the identity: where , i.e.

Let

Substitution:

Bounds:

Let

Substitution:

Bounds:

Let

1. Evaluate

Use the identity: where , i.e.

Let

1. If and when , then find in terms of .

Use the identity: where , i.e.

Alternatively, consider the double angle result for cosine and rearrange.

Substitute

Substitute

Integrating inverse trigonometric functions

Practice is needed in the use of the product rule, quotient rule, and chain rule in relation to the inverse trigonometric functions. Students will need practice on questions involving the inverse trigonometric functions and integration by substitution.

1. Differentiate , and hence show that .

Let

This implies that the function is a constant. To investigate the value of this constant:

Let

If

Then

as required.

1. If , find and show that for all in the domain.

This implies that the function is a constant. To investigate the value of this constant:

Now, let

as required.

Let

Let



Let

Other questions

1. Prove that . Hence use the substitution to show that

Use the product rule to differentiate

i.e. , where:

as required.

Given the above result,

Let

Substitution:

Bounds:

as required.