 Applications of the modified exponential growth and decay model

Activity 1: Newton’s law of cooling

1. Boil a jug of water and pour an equal amount of liquid into 3 cups.
2. Place one cup in the classroom, one in the fridge and one in the freezer.

Note: You want the temperature to be constant in each environment. You could achieve this by using a cooler or heater in the classroom and a temperature controller on the fridge or freezer to regulate the temperature.

1. Record the temperature of each cup initially and then every 10 minutes.
2. Record the ambient temperature of the fridge, freezer and room (P)

| Elapsed time (minutes) | Temperature ($℃$)Classroom | Temperature ($℃$)Fridge | Temperature ($℃$)Freezer |
| --- | --- | --- | --- |
| 0 |       |       |       |
| 10 |       |       |       |
| 20 |       |       |       |
| 30 |       |       |       |
| 40 |       |       |       |
| 50 and so on |       |       |       |

For each situation (Classroom, Fridge or Freezer),$T=P+Ae^{kt}$

* Determine the value of k. (Refer to Resource: another-way-to-determine-k.DCOX page 3)
* Determine the equation relating the temperature of the water to the elapsed time.
* Determine the rate of change of the temperature with respect to time. (dT/dt)
* Construct a scatterplot of the data and add the equation (use Geogebra or other graphing software)
* Predict the temperature of the water after 35 minutes.
* Predict when the temperature will fall below 30($℃$) correct to the nearest minute.

Activity 2: Estimating the time of death of a potato

Teacher note: Prior to class, heat the potato to a temperature of 90$℃$*,* place it in the classroom, noting the time it was removed from the oven. Don’t disclose this until the end of the activity.

Coroners can use several measurements of the temperature of a deceased person to estimate the time of death of the individual.

This activity will demonstrate how this method works using a potato.

The potato will represent our ‘body’. For our scenario, a ‘live’ potato has a body temperature of 90 $℃$.

1. Record the ambient room temperature $℃$ (P)
2. Take two temperature readings of the potato and record below:

| Time (for example 9.40am) | Temperature $℃$ |
| --- | --- |
|       |       |
|       |       |

1. Consider the equation $T=P+Ae^{kt}$ where t is the time after the potato died in minutes. Substitute the initial body temperature and the room temperature (P) to determine the value of A.
2. Use the two temperature readings from part b to produce two equations using the formula $T=P+Ae^{kt}$ Hint: Let your two t values be$t\_{1}$ and $t\_{1}+30 $(if your readings were 30 minutes apart)
3. Solve these equations simultaneously to find k.
4. Write a new equation of the form $T=P+Ae^{kt}$(You should have determined the constants P, A and k)
5. Substitute one of your readings from part b to determine its actual t value (time after death).
6. Determine what time the potato died to the nearest minute. (for example: 8:31 am)
7. Compare the estimated time the potato died with its true time of death (when the potato was removed from the oven)

Activity 3: Problem solving questions

Questions modified from [AMSI](https://www.amsi.org.au/ESA_Senior_Years/PDF/GrowthDecay3e.pdf)

Question 1: Ice cream

Background information:

* It is a hot summer’s day. The temperature outside is 40$℃$.
* You decide to buy an ice-cream.
* It comes out of a freezer which is kept at -22$℃$.
* After one minute, the ice-cream has warmed to -5$℃$.

What is the temperature of the ice-cream after 7 minutes?

Question 2: Coffee

Background information:

* It is a cool winter’s day. The temperature inside your house is 18$℃.$
* You decide to make a cup of coffee with boiling water (100$℃$)
* After three minutes it has cooled to 75$℃$.

What is its temperature after 6 minutes?

When will the coffee drop below 38$℃$ and taste cold?

Question 3: Heated metal

Background information:

* You are given a very hot sample of metal, and wish to know its temperature.
* You have a thermometer, but it only measures up to 200$℃$ and the metal is currently too hot to measure.
* You leave the metal in your workshop which is at a constant temperature of 25$℃$.
* After 6 minutes the temperature was 140$℃$.
* After another 4 minutes the temperature was 90$℃$.

What was the initial temperature of the metal?

Activity 4: Is the standard growth or modified growth model better for a given scenario?

* Revisit the data from activities 1, 2 and 3 in Resource: applications-of-exponential-growth-and-decay.DOCX
* Refer to Resource: another-way-to-determine-k.DOCX and fit a standard exponential model and a modified exponential model to each problem.
* Graph the algebraic models verse the scatterplot of the data.
* Which model is the best fit?