 Year 11 mathematics advanced

| MA-T2 Trigonometric functions and identities | Unit duration |
| --- | --- |
| The topic Trigonometric Functions involves the study of periodic functions in geometric, algebraic, numerical and graphical representations. A knowledge of trigonometric functions enables the solving of practical problems involving triangles or periodic graphs, such as waves and signals. The study of trigonometric functions is important in developing students’ understanding of periodic behaviour, a property not possessed by any previously studied functions. Utilising this property, mathematical models have been developed that describe the behaviour of many naturally occurring periodic phenomena, such as vibrations or waves, as well as oscillatory behaviour found in pendulums, electric currents and radio signals. | 2 weeks |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is to use trigonometric identities and reciprocal relationships to simplify expressions, to prove equivalences and to solve equations. Students develop their ability to prove identities, simplify expressions and solve trigonometric equations. Trigonometric expressions and equations provide a powerful tool for modelling quantities that vary in a cyclical way such as tides, seasons, demand for resources, and alternating current. The solution of trigonometric equations may require the use of trigonometric identities. | A student:uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identitiesMA11-4uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8provides reasoning to support conclusions which are appropriate to the context MA11-9 |

| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| Students should have studied Stage 5.2 right-angled triangles, Stage 5.3 trigonometry and Pythagoras’ theorem, algebraic techniques, equations and properties of geometrical figures. | * After learning reciprocal trigonometric graphs, students given 10 minutes to sketch all trigonometric graphs they have learnt in the topic
* Students to complete the [30 minute revision session](http://mathslinks.net/faculty/revision-checklist-for-nsw-stage-6-mathematics-2-unit) for Trigonometry from mathslinks.net
* Students to complete the [Maths Links revision booklet](http://mathslinks.net/faculty/2u-trigonometry-notes) without notes to identify key areas needed for revising
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All outcomes referred to in this unit come from [Mathematics Advanced](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017) Syllabus
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Glossary of terms

| Term | Description |
| --- | --- |
| identity  | An identity is a statement involving a variable(s) that is true for all possible values of the variable(s). |
| radian | One radian is the angle subtended at the centre of a circle by an arc of 1 unit in length and the radius of 1 unit in length. |
| periodicity | The quality of recurring at regular intervals. |

Lesson sequence

| Lesson sequence | ContentStudents learn to: | Suggested teaching strategies and resources  | Date and initial | Comments, feedback, additional resources used |
| --- | --- | --- | --- | --- |
| Reciprocal trigonometric functions(1 lesson) | * define the reciprocal trigonometric functions, , and
 | Defining the reciprocal trigonometric functions* Students to revise the trigonometric ratios for and radians prior to starting this unit of work
* Introduce and define the reciprocal trigonometric functions:

**Note** – cotangent can also be define as * The reciprocal trigonometric functions can also be defined as:
* The reciprocal ratios should also be connected to the unit circle, where:
	+

The connection to the unit circle is clearly explained on [Bright Storm](https://www.brightstorm.com/math/trigonometry/trigonometric-functions/the-reciprocal-trigonometric-functions/) (duration 2:15).* Appropriate abbreviations should be shown for the reciprocal trigonometric functions. It should be noted that in modern usage, csc is an appropriate abbreviation for cosecant.

**NESA exemplar questions*** Find exact values of:
	+
	+
 |  |  |
| Sketching reciprocal trigonometric functions(1 lesson) | * sketch the graphs of reciprocal trigonometric functions in both radians and degrees
 | **Sketching the reciprocal trigonometric functions*** Revise the graphs of sine, cosine and tangent in both radians and degrees. Students should be prompted to find out the values of … if required.
* Students should develop the sketches of the reciprocal trigonometric functions from the original trigonometric functions. This should include identifying properties of those graphs such as asymptotes, periodicity, maximum and minimum values and their relationship to the graphs of the trigonometric functions, noting that maximum and minimum values are local not global for and
* Utilise graphing software templates to assist in developing the graphs. [Reciprocal trigonometry](https://www.geogebra.org/m/Mkf9fE4c) has a slider to develop the graph of and can be adjusted to graph .
* Once the graphs are developed and defined using both radians and degrees, student should be exposed to questions that require them to sketch two trigonometric functions on the same set of axes and find the points of intersection. These questions should allow students to graph using technology, rather than by hand for accuracy.

NESA exemplar question* On the same set of axes, sketch and for . Using graphing technologies or otherwise, find the values of for which .
 |  |  |
| Pythagorean and Trigonometric identities(1 – 2 lessons) | * use provided that
* prove and apply the Pythagorean identities , and  (ACMSM046)
* prove trigonometric identities
 | Exploring trigonometric identities* The result that and all values on the unit circle should be revised, as well as the reciprocal trigonometric functions, prior to commencing the exploration of further trigonometric identities.
* Trigonometric identities can be used to prove statements and to assist in solving equations. Students should have the opportunity to observe the difference between an equation and an identity, noting that an equation involves solving to find the value(s) of a pronumeral, whereas an identity is proving one side (LHS) is equal to the other side (RHS).
* Make explicit that represents
* The Pythagorean identity, , must be proved. [Interactive Mathematics](https://www.intmath.com/analytic-trigonometry/trig-ratios-interactive.php) website can be used as an introduction. It has a slider on a unit circle that shows the values of , and then demonstrates that the sum of the values squared always equals one, no matter where you are on the unit circle. [MathIsFun](https://www.mathsisfun.com/algebra/trigonometric-identities.html) website explores the complete proof.
* Once the first Pythagorean identity is proved, students should be shown how to obtain the other two identities:
	+ If you divide all terms by , , you get: .
	+ If you divide all terms by , , you get: .
* Once all identities are proved, students can now be shown how to apply these to proofs. When proving, ensure that students correctly set out a formal proof, using LHS and RHS and working down their page.
* A common mistake is students stating the identity to be proven as a fact and then attempting to manipulate both sides at the same time, as well as adding/subtracting/multiplying/dividing terms to each side. The correct process is to begin with one side only and using the identities to show that it is equal to the other, working down the page and not across.

NESA exemplar questions* Show that:
* Prove that . Hence prove that .
 |  |  |
| Evaluating trigonometric expressions(1 lesson) | * evaluate trigonometric expressions using angles of any magnitude and complementary angle results
 | **NESA exemplar questions*** Given that , and that . Find the values of:
	+
	+ .
* If , express in terms of :

Given a trigonometric ratio and a domain, find another trigonometric ratio* Students can sketch a triangle on the unit circle with the trigonometric function they are given and in the quadrant stated by the domain. They may need to use their knowledge of reciprocal trigonometric ratios to ensure the values are placed on the correct sides of the right-angled triangle.
* Students may need to use Pythagoras’ theorem to find the length on the third side of the triangle, so they can find the required trigonometric ratio from the given angle.
 |  |  |
| Solving trigonometric equations(1 lesson) | * simplify trigonometric expressions and solve trigonometric equations, including those that reduce to quadratic equations
	+ know the difference between an equation and an identity
 | Solving trigonometric equations* It would be useful to begin by connecting all relevant previous learning to this component of the unit. This should include solving quadratic equations, ensuring that students remember to first make the equation equal to zero; evaluating expressions using angles of any magnitude, exact values and trigonometric graphs and considering each of the trigonometric identities.
* Start with simple trigonometric equations such as and  for or . Use the exact values to demonstrate that the related reference angle is 30 or for both the above questions. Use to the unit circle and utilise knowledge of angles of any magnitude to solve the trigonometric equations as having the following solutions:

or or* Students should work through similar questions that rotate around different boundary angles both in the positive and negative direction. Students should also examine what happens when the domain extends beyond 360 or below -360.
* Students should be exposed to questions where they are required to manipulate the given domain in the question per the trigonometric angle. For example, . The domain becomes by multiplying the given domain by 2.

Similarly, in , the domain becomes by adding 20 to the given domain.* Students will then work through solving trigonometric equations that combine all of the above information and involve manipulating the original given equation to make the trigonometric ratio the subject. For example, for the domain -360°≤x≤360°.
* Students should also be exposed to trigonometric equations that can be reduced to quadratic equations.

**NESA exemplar questions** * Solve for
* Express in terms of and hence solve the equation for .
 |  |  |

Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘comments, feedback, additional resources used’ section.