 Year 12 Mathematics Advanced

| MA-M1 Modelling financial situations | Unit duration |
| --- | --- |
| The topic Financial Mathematics involves sequences and series and their application to financial situations. A knowledge of financial mathematics enables analysis and interpretation of different financial situations, the calculation of the best options for the circumstances, and the solving of financial problems. The study of financial mathematics is important in developing students’ ability to make informed financial decisions, to be aware of the consequences of such decisions, and to manage personal financial resources prudently. | 7 weeks |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is the meaning and mathematics of annuities, including the introduction of arithmetic and geometric sequences and series with their application to financial situations. Students develop an understanding for the use of series in the borrowing and investing of money, which are common activities for many adults in contemporary society. Annuities represent financial plans involving the sum of a geometric series and can be used to model regular savings plans, including superannuation. Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students. | A student:* models and solves problems and makes informed decisions about financial situations using mathematical reasoning and techniques MA12-2
* applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems MA12-4
* chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
* constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10
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| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| This topic builds upon the financial mathematics concepts explored in Stage 5. | Formative assessment: The investigation style activities provide students with opportunities to reason and communicate through “what if?” style questions; and staff opportunities to gauge their understanding. The independent activities within this unit should be used to assess students’ fluency and problem solving. |

All outcomes referred to in this unit come from [Mathematics Advanced](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017) Syllabus
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Glossary of terms

| Term | Description |
| --- | --- |
| annuity | An annuity is a compound interest investment from which payments are made or received on a regular basis for a fixed period of time. |
| arithmetic sequence | An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. |
| arithmetic series | An arithmetic series is a sum whose terms form an arithmetic sequence. |
| future value | The future value of an investment or annuity is the total value of the investment at the end of the term of the investment, including all contributions and interest earned. |
| future value interest factors | Future value interest factors are the values of an investment at a specific date. A table of these factors can be used to calculate the future value of different amounts of money that are invested at a certain interest rate for a specified period of time. |
| geometric sequence | A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the common ratio. |
| geometric series | A geometric series is a sum whose terms form a geometric sequence. |
| partial sum | The sum of part of a sequence. |
| present value | The present value of an investment or annuity is the single sum of money (or principal) that could be initially invested to produce a future value over a given period of time. |
| series | A series is the sum of the terms of a particular sequence. |
| sequence | In mathematics, a sequence is a set of numbers whose terms follow a prescribed pattern. Mathematical sequences include arithmetic sequences and geometric sequences. |

| **Lesson sequence** | **Content** | **Suggested teaching strategies and resources**  | **Date and initial** | **Comments, feedback, additional resources used** |
| --- | --- | --- | --- | --- |
| Modelling compound interest and annuities(2-3 lessons) | **M1.1: Modelling investments and loans*** solve compound interest problems involving financial decisions, including to a home loan, a savings account, a car loan or superannuation **AAM**
	+ identify an annuity (present or future value) as an investment account with regular, equal contributions and interest compounding at the end of each period, or a single-sum investment from which regular, equal withdrawals are made
	+ use technology to model an annuity as a recurrence relation and investigate (numerically or graphically) the effect of varying the interest rate or the amount and frequency of each contribution or a withdrawal on the duration and/or future or present value of the annuity
	+ use a table of interest factors to perform annuity calculations, eg calculating the present or future value of an annuity, the contribution amount required to achieve a given future value or the single sum that would produce the same future value as a given annuity
 | **Resources to use throughout the topic:*** Refer to the [student reference sheet](https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017) for appropriate formulae.
* [Moneysmart](https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/investment-calculators) investment calculators to check solutions for financial problems

**Modelling compound interest and annuities*** Introduction to the topic. Students to brainstorm how people interact with a bank and group these as either an investment or loan.
* Optional review of simple and compound interest, modelling these as a recurrence relation using a spreadsheet.

Teacher to define annuity, present value, future value (see glossary of terms). Students to model and investigate annuities as recurrence relations using technology. Students need to be able to explain the effect of varying the interest rate, contribution (or withdrawal), frequency of the contribution (or withdrawal) on the duration and/or future or present value of the annuity. **Resources:** investigation-m1-1.DOCX, investigation-m1-1.XLSX* Define future value interest factors and model using a [table of future value interest factors](https://www.accountingtools.com/articles/2017/5/17/future-value-of-an-ordinary-annuity-table?rq=future%20value) to perform annuity calculations. **Resources:** fv-interest-factors.DOCX, fv-interest-factors.XLSX
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| Introduction to sequences and series and the th term of an arithmetic sequence (1-2 lessons) | **M1.2: Arithmetic sequences and series*** know the difference between a sequence and a series
* recognise and use the recursive definition of an arithmetic sequence: **AAM**
* establish and use the formula for the th term (where is a positive integer) of an arithmetic sequence: , where is the first term and is the common difference, and recognise its linear nature **AAM**
 | **Introduction to sequences and series and the nth term of an arithmetic sequence*** Introduction: Teacher to define key terms: sequence, series, term, partial sum
* Students can complete basic questions to illustrate understanding of the terminology.
	+ is a \_\_\_\_\_\_\_\_\_\_ of odd numbers
	+ is a \_\_\_\_\_\_\_\_\_ of odd numbers
	+ Find the sum of the first 5 terms of the series
	+ Find the 8th term of the sequence
* Arithmetic and geometric sequences and series:

Students to recognise two specific styles of series and/or sequences by discussing the rule to obtain the next term:Teacher to define:* + Arithmetic sequences and arithmetic series
	+ Geometric sequences and geometric series

Refer to the glossary of terms and the patterns above. Optional: consider arithmetic patterns as being formed through an additive process and geometric patterns through a multiplicative process.* Terms of an Arithmetic sequence:
	+ Teacher to define: , where is the nth term. To find the subsequent term in an arithmetic sequence, add a common difference.

Note: is a positive integer* + Discuss sequences where or when . E.g. identify the common difference as being negative for a decreasing arithmetic sequence.
* The nth term of an arithmetic sequence
	+ Teacher to define as the first term.
	+ Students to discover the nth term of an arithmetic sequence in terms of and .

**Resources:** nth-term-arithmetic-sequence.DOCX, nth-term-arithmetic-sequence.XLSX* + Teacher to conclude: , where is the nth term, is the first term and is the common difference.
* Guided practice:
	+ Teacher to model solving questions involving the terms of arithmetic series and sequences. Sample questions are included in part 1 of:

**Resource:** arithmetic-sample-questions.DOCX* + Note: The use of sigma notation is referred to in NESA’s sample unit.

Example: Use of sigma notation to represent Method: identify and that there are terms by solving This leads to the solution  |  |  |
| The sum of n terms in an arithmetic sequence or series (1-2 lessons) | * establish and use the formulae for the sum of the first terms of an arithmetic sequence:  where is the last term in the sequence and **AAM**
 | **The sum of n terms in an arithmetic sequence or series*** Teacher to define as the last term of an arithmetic sequence.
* Teacher to lead the establishment of the formulae for the sum of the first terms of an arithmetic sequence: and
	+ Sample methods:

**Resource:** sum-of-an-arithmetic-sequence.DOCX* + Visual representation:

**Resource:** visual-of-arithmetic-sum.GGB* Teacher to model solving questions involving the sum an arithmetic sequence, see part 2 of:

**Resource:** arithmetic-sample-questions.DOCX |  |  |
| Solving problems involving arithmetic sequences and series(1-2 lessons) | * identify and use arithmetic sequences and arithmetic series in contexts involving discrete linear growth or decay such as simple interest (ACMMM070) **AAM**
 | **Solving problems involving arithmetic sequences and series*** Teacher to model solving problems in contexts involving discrete linear growth or decay using an arithmetic sequence or series.

Teacher to model defining Tn for a given problem. Example: T1 may represent the balance of a simple interest account at the end of period 1.* Sample questions include:
	+ Simple interest investment
	+ Withdrawing money from a trust account
	+ A piggy bank
	+ Building a block tower
	+ Seats in a theatre
	+ Dropping a ball from a tower

**Resource:** arithmetic-sample-questions.DOCX |  |  |
| Introduction to geometric sequences and the th term of a geometric sequence (1-2 lessons) | **M1.3: Geometric sequences and series*** recognise and use the recursive definition of a geometric sequence: (ACMMM072) **AAM**
* establish and use the formula for the th term of a geometric sequence: , where is the first term, is the common ratio and is a positive integer, and recognise its exponential nature (ACMMM073) **AAM**
 | **Introduction to geometric sequences and the th term of a geometric sequence*** Teacher to review the definition of a geometric sequence and series.
* Terms of an geometric sequences:

Teacher to lead the exploration of a range of geometric sequences such as: For each, consider: * + If I have the 1st term, what do I multiply it by to get the 2nd?
	+ If I have the 2nd term, what do I multiply it by to get the 3rd?
	+ If I have the 3rd term, what do I multiply it by to get the 4th?
	+ If I have the th term, , what do I multiply it by to get the nth term, ?

Develop the recursive definition for a geometric sequence * + Define r as the as the common ratio.
	+ Teacher to discuss where , . Refer to samples above.
	+ Teacher to question, imagine the first term is 5 and the common ratio is 1. What will the 2nd term be? The 3rd?
* The nth term of a geometric sequence:
	+ Students to discover the nth term of a geometric sequence in terms of and .

**Resource:** nth-term-geometric-sequence.DOCX* + Teacher to conclude: , where is the first term, is the common ratio and is a positive integer and refer to its exponential nature.
* Guided practice: Teacher to model solving questions involving the terms of geometric series and sequences. For sample questions see part 1 of:

**Resource:** geometric-sample-questions.DOCX |  |  |
| The sum of n terms in a geometric series (1-2 lessons) | * establish and use the formula for the sum of the first terms of a geometric sequence:  (ACMMM075) **AAM**
 | **The sum of n terms in a geometric series*** Teacher to lead the establishment of the formulae for the sum of the first n terms of an geometric sequence:
	+ Sample method to establish the formula:

**Resource:** sum-of-a-geometric\_sequence.DOCX* + Discuss when it is appropriate to use each formula and recognise both formulas will work if applied correctly.
* Guided practice:
	+ Teacher to model solving questions involving the sum of the terms of geometric series and sequences. For sample questions, see part 2 of:

**Resource:** geometric-sample-questions.DOCX |  |  |
| Limiting sum of a geometric series(1 lesson) | * derive and use the formula for the limiting sum of a geometric series with : **AAM** Critical and creative thinking icon
	+ understand the limiting behaviour as and its application to a geometric series as a limiting sum
	+ use the notation for
 | **Limiting sum of a geometric series*** Define an infinite geometric sequence as a geometric sequence with infinite terms.
* Students to use a spreadsheet to examine what happens as n increases for two geometric sequences, where and where .

Students to observe:* + When , as ,
	+ When , as ,

**Resource:** infinite-geometric-sequence.XLSXStudents to consider* + Can we calculate the sum if ?
	+ Can we calculate the sum if ?
* Teacher to define the limiting sum of a geometric series and derive the formula with :
* Guided practice:
	+ Teacher to model solving questions involving the limiting sum of geometric series and sequences. For sample questions, see part 3 of:

**Resource:** geometric-sample-questions.DOCX |  |  |
| Solving problems involving compound interest (1-2 lessons) | **M1.4: Financial applications of sequences and series*** use geometric sequences to model and analyse practical problems involving exponential growth and decay (ACMMM076) **AAM**
	+ calculate the effective annual rate of interest and use results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly (ACMGM095)
	+ solve problems involving compound interest loans or investments, for example determining the future value of an investment or loan, the number of compounding periods for an investment to exceed a given value and/or the interest rate needed for an investment to exceed a given value (ACMGM096)
 | **Solving problems involving compound interest*** Effective annual rate of interest
	+ Teacher to define an effective annual rate of interest.
	+ Teacher to model calculating an effective annual rate of interest.
	+ Students to compare investment returns and the cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly.

**Resource:** effective-annual-rate-of-interest.DOCX* Compound interest loans and investments

Teacher to model:* + establishing the meaning of , the balance owing of the loan or value of the investment
	+ calculating the future value after a given time period
	+ calculating the number of compounding periods required to exceed a given value. Given and , solve the following for :
	+ calculating the interest rate needed to exceed a given value. Given and , solve the following for :

Students to solve related problems. |  |  |
| Solving problems involving time repayments (1-2 lesson) | * use geometric sequences to model and analyse practical problems involving exponential growth and decay (ACMMM076) **AAM**
	+ recognise a reducing balance loan as a compound interest loan with periodic repayments, and solve problems including the amount owing on a reducing balance loan after each payment is made
* solve problems involving financial decisions, including a home loan, a car loan **AAM**
 | **Solving problems involving time repayments*** Recognise a reducing balance loan as a compound interest loan with periodic repayments.
* Teacher to model the loan using a geometric series and then:
	+ Calculating the balance owing on a loan
	+ Calculating the periodic repayment on a loan
	+ The total repaid
	+ The total interest paid

Example of calculations for a home loan: **Resource:** modelling-home-loans.DOCX* Students to solve problems related to home loans, car loans and other financial situations. For example students may:
	+ For a home loan, compare the repayments, total repaid and/or interest paid for a range of interest rates.
	+ Choose a car they want to purchase, research interest rates on car loans, calculate the repayment per period and check their answers with online calculators.
	+ Choose a house they want to purchase, research interest rates on home loans, calculate the repayment per period and check their answers with online calculators.
	+ Research median house prices in their area, complete the above activity using this as the home’s value.

Students will need to consider any required deposit.* Students can model situations where the interest rate changes after a certain time period.
 |  |  |
| Solving problems involving savings and superannuation(1-2 lesson) | * solve problems involving financial decisions, including a savings account, or superannuation **AAM**
	+ calculate the future value or present value of an annuity by developing an expression for the sum of the calculated compounded values of each contribution and using the formula for the sum of the first terms of a geometric sequence
	+ verify entries in tables of future values or annuities by using geometric series
 | **Solving problems involving savings and superannuation*** Review the definition of an annuity.
* Calculations involving present/future values include:
	+ Given a contribution, calculate the balance of a savings or superannuation account at a future date (or retirement).
	+ What contribution is needed per period to achieve a set savings or superannuation account balance at a future date (or retirement)?
	+ What balance is required (present value) to produce a set regular withdrawal in retirement?
	+ Given a superannuation balance at retirement (present value), what regular withdrawal can be made during retirement?
	+ Scenarios where the contribution or interest rate changes. See NESA’s [Financial Mathematics Carousal questions](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017) under sample units.

To make problems contextually relevant, students can research typical returns on superannuation accounts, life expectancies, retirement ages and incomes to consider employer contributions.Contributions or withdrawals with annuities are at the end of the period unless noted.* Verify entries in a table of future values or annuities using a geometric series.
	+ Students find an online [present](https://www.accountingtools.com/articles/2017/5/16/present-value-of-an-ordinary-annuity-table)/[future](https://www.accountingtools.com/articles/2017/5/17/future-value-of-an-ordinary-annuity-table?rq=future%20value) value of an annuity table and verify results.
	+ Students can construct their own tables in a spreadsheet to confirm.
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Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.