 Year 12 Mathematics Advanced

| MA-C2 Differential calculus | Unit duration |
| --- | --- |
| The topic Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It involves the development of two key aspects of calculus, namely differentiation and integration. The study of calculus is important in developing students’ capacity to operate with and model situations involving change, using algebraic and graphical techniques to describe and solve problems and to predict outcomes in fields such as biomathematics, economics, engineering and the construction industry. | 4 weeks |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is to develop and apply rules for differentiation to a variety of functions. Students develop an understanding of the interconnectedness of topics from across the syllabus and the use of calculus to help solve problems from each topic. These skills are then applied in the following subtopic on the second derivative in order to investigate applications of the calculus of trigonometric, exponential and logarithmic functions. | A student:   * applies calculus techniques to model and solve problems MA12-3 * applies appropriate differentiation methods to solve problems MA12-6 * chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9 * constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10 |

| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| The material in this topic builds on content from the Year 11 topics of MA-C1 Introduction to differentiation and MA-E1 Exponential and logarithmic functions. It would also be useful to have completed the Year 12 topic of MA-T3 Trigonometric functions and graphs. | * Formative assessment: Students to use both online graphing tools and pen-and-paper methods to demonstrate informal and, where appropriate, formal investigations and proofs for the concepts explored in this topic. |

All outcomes referred to in this unit come from [Mathematics Advanced](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017) Syllabus  
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Glossary of terms

| Term | Description |
| --- | --- |
| exponential growth and decay | Exponential growth occurs when the rate of change of a mathematical function is positive and proportional to the function’s current value. Exponential decay occurs in the same way when the growth rate is negative. |
| limit | The limit of a function at a point , if it exists, is the value the function approaches as the independent variable approaches .  The notation used is:  This is read as ‘the limit of as approaches is ’. |

| **Lesson sequence** | **Content** | **Suggested teaching strategies and resources** | **Date and initial** | **Comments, feedback, additional resources used** |
| --- | --- | --- | --- | --- |
| Establishing the derivatives for and  (1 or 2 lessons) | C2.1**: Differentiation of trigonometric, exponential and logarithmic functions**   * establish the formulae and by numerical estimations of the limits and informal proofs based on geometric constructions (ACMMM102) | **Investigating the limits of**   * Staff need to determine early that all calculations involving trigonometric functions in calculus are performed using radians and will not be accurate if degrees are used. * Students need to establish the limit, which represents the gradient of at . * Students need to calculate the expressions, in radians, for values of that approach 0, for example, the values of when . * Leading to the numerical estimation of and an estimate for the gradient of the curve at the point * A similar investigation can be performed for  as both sin x, tan x and x converge at the origin creating an unusual anomaly which can be approximated to the calculation of .   **Informal construction of the gradient of**   * From the curve of and using the limit established above, establish the gradient of the curve at points where it can be determined   Graph of y=sin(x) between 0 and 2π. The graph has peak value of 1 at x = π/2 and minimum value of -1 at  x = 3π/2.”   * Leading to the informal results:   when ,  when ,  when ,  when ,  when ,   * Sketch these points and lead students to the result for * A similar investigation can be structured to investigate the gradient of * Staff may like to use the formal method of differentiation from first principles to establish the results above but this is not a syllabus requirement. * Staff may like to use this [Geogebra app to investigate the gradient](https://www.geogebra.org/m/zdcrt4zh) of the curve |  |  |
| Finding derivatives of expressions involving and  (2 or 3 lessons) | C2.1**: Differentiation of trigonometric, exponential and logarithmic functions**   * calculate derivatives of trigonometric functions   C2.2: **Rules of differentiation**   * apply the product, quotient and chain rules to differentiate functions of the form  and where and are any of the functions covered in the scope of this syllabus, for example and (ACMMM106)   + use the composite function rule (chain rule) to establish and use the derivatives of , and ) | **Finding derivatives of expressions involving and**   * Staff need to deliver questions of the form * or where k is constant * or where k is constant * Expressions involving a mixture of these terms, polynomial terms and exponential terms. * And expressions that require differentiation using the product, quotient and chain rules. * Students need to explore derivatives in the form where is a function of by applying the chain rule. Lead students to the generalised result . Students need to run a similar investigation into * Lead students to use the quotient rule to determine the derivative by using the identity . * Use this result to lead students to apply the chain rule to determine |  |  |
| Differentiating exponential functions  (2 or 3 lessons) | C2.1: **Differentiation of trigonometric, exponential and logarithmic functions**   * establish and use the formula   + using graphing software or otherwise, sketch and explore the gradient function for a given exponential function, recognise it as another exponential function and hence determine the relationship between exponential functions and their derivatives   C2.2: **Rules of differentiation**   * apply the product, quotient and chain rules to differentiate functions of the form  and where and are any of the functions covered in the scope of this syllabus, for example and (ACMMM106)   + use the composite function rule (chain rule) to establish that | **Differentiating exponential functions**   * Lead students to apply the chain rule to determine * Review and establish the result to be used within the proof below. * Structure a formal proof by starting with the LHS of the identity   Note, if then can be expressed as   * Students need to answer questions in the form   and expressions that include exponential terms as part of product, quotient and chain rule questions.   * Students need to extend their differentiation skills developed and apply them to find the equation of tangents and normal at various points to the curve. * Staff and students may like to use this [Geogebra app to demonstrate curves](https://www.geogebra.org/m/vwdcekey) of the function and its derivative function. * Staff may like to reference the Exponential and Logarithmic Functions (PDF) resource published by the [Australian Mathematical Sciences Institute](https://amsi.org.au/ESA_Senior_Years/SeniorTopic3/3_md/SeniorTopic3h.html). Page 16 refers to this concept. |  |  |
| Finding the derivative of logarithmic functions  (2 or 3 lessons) | C2.1: **Differentiation of trigonometric, exponential and logarithmic functions**   * calculate the derivative of the natural logarithm function * establish and use the formula   C2.2**: Rules of differentiation**   * apply the product, quotient and chain rules to differentiate functions of the form  and where and are any of the functions covered in the scope of this syllabus, for example and (ACMMM106)   + use the composite function rule (chain rule) to establish that   + use the logarithmic laws to simplify an expression before differentiating | **Finding the derivative of logarithmic functions**   * Staff may like to lead students to the result   Let  or   * Staff need to establish the result   Proof  using the change of base logarithmic law  Students need to answer questions in the form   * [How many ways can the solution be generated? Using the chain rule? Or Logarithmic laws?] * And expressions that include logarithmic terms as part of chain, product and quotient rule questions. * Students need to explore differentiation questions of the form where is some function of . By analysing the results lead them to develop the generalised expression. * Students need to extend their differentiation skills developed and apply it to find the equation of tangents and normal at various points to the curve. |  |  |

Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.