 Year 12 Mathematics Advanced

**Assessment task:** MA-T3 Trigonometric functions and graphs

**Driving question:** Can mathematics predict periodic phenomena?

Outcomes

* **MA12-1** uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts
* **MA12-5** applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs
* **MA12-9** chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use
* **MA12-10** constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

This document references the [Mathematics Advanced Syllabus](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017) © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2017.

Learning across the curriculum

* Critical and creative thinking
* Ethical understanding
* Information and communication technology capability
* Literacy
* Numeracy
* Work and enterprise

Context

Students are to investigate periodic phenomena and model them with trigonometric functions and their graphs.

There are three key objectives for this task

* Students understand how to examine a set of data from an observable periodic phenomena and identify trends applicable to a sine or cosine function.
* Students understand how to fit, evaluate and adjust a model used to represent periodic phenomena.
* Students analyse a model to make inferences or predictions.

Background

During this task students should use graphing software, such as Geogebra or Desmos, to model and predict periodic phenomena.

Students may like to refer to the reference document Modelling-of-periodic-phenomena.DOCX and Applications-of-periodic-phenomena.DOCX for parts 2 and 3.

Task

Part 1 – sound waves

The shape of sound waves will determine the properties of the sound produced and how it is perceived by humans.

1. The volume of sound is proportional to the amplitude of the sound wave assuming they have the same frequency.

Students are to construct and label two trigonometric functions and their corresponding graphs to demonstrate a high and low volume of sound.

Students are to state the properties of the trigonometric functions. i.e. The period, frequency and amplitude.

1. The higher the frequency, the higher the pitch of the sound wave.

Students are to construct and label two trigonometric functions and their corresponding graphs to demonstrate a high and low pitch with a consistent amplitude.

Students are to state the properties of the trigonometric functions. For example – the period, frequency and amplitude.

Part 2 – ocean tides

Ocean sea levels vary between a changing high and low tide periodically as a result of the location of the moon and its position relative to the sun.

Students are to:

1. Collect tidal data:
   * Choose a location (Example: Forster, NSW)
   * For this location, students are to collect 7 days of historical low and high tide data (time and height). Many websites contain tidal data. [Willyweather](https://tides.willyweather.com.au/) has a graph and table of historical and future high and low tides.

A suggested table for recording this information is:

| Date | Time | Elapsed time (hours) | Height (m) |
| --- | --- | --- | --- |
|  |  |  |  |

Note: Elapsed time refers to the number of hours since the first recording. For example, 5 hours and 15 minutes would be recorded as 5.25.

1. Model 1: Model the scenario using a single trigonometric function:
   * Plot the height verse elapsed time in a suitable graphing software. Suggested software includes Geogebra and Desmos.
   * Interpret the observable amplitude, vertical shift and period of the data set, to fit a trigonometric model to the data. Provide reasons for your choices.
2. Model 2: Model the scenario using multiple functions, one or more of which must be a trigonometric function:
   * By examining data points or trends not captured by the simple model, establish another model using a combination of functions. Justify your model using appropriate mathematical calculations. Refer to Method 2 and/or 3 in Modelling-of-periodic-phenomena.DOCX
3. Predictions: Use your models to predict the high and low tide times and heights on a future date. A suggested table for recording this information is:

| Low Tide | Date | Time | Height |
| --- | --- | --- | --- |
| Model 1 |  |  |  |
| Model 2 |  |  |  |
| Actual |  |  |  |

Note: A similar table would be required for high tides.

1. Chose a model: Select the more accurate model, justifying your choice by comparing and contrasting the pros and cons of each model with reference to the original data, predictions and any observed limitations or potential for improvements.
2. [Fishing monthly](http://www.fishingmonthly.com.au/Articles/Display/20223-Best-fishing-times-and-tides-weighing-it-all-up) contained information about the best times to fish with regards to tides.

Using your selected future date and model, find a time(s) or time range(s) corresponding to each of the following scenarios:

* + Feeding tends to slow when the tide changes.
  + Flathead tend to feed during falling water or the first hour of an incoming tide.
  + Bream tend to be present during high tide around mangrove flats.
  + 1 hour before and after a low tide is a good time to catch whiting off a beach
  + Predator fish like a running tide but avoid the fastest flowing water.
  + At what time(s) is the maximum rate of change in the height of the tide?
  + What is the rate of change in the height of the tide at this time?

Part 3 – commercial trends

1. Select a trend that follows a cyclical cycle. You may like to consider food items that are more commonly eaten during a portion of the year such as stew or ice cream.
2. Collect five years of data using google trends for Australia. [Google Trends](https://trends.google.com/trends)
3. Model your data using a trigonometric function or combination of functions.
4. Explain your choice of model and identify any limitations and possible improvements.
5. Repeat steps 2-5 for a country in the northern hemisphere.
6. For each location, use your model to predict the maximum and minimum interest in your topic in 2023.
7. Explain any differences or similarities in the models and corresponding predictions.
8. When in 2023 will the interest in your trend be at 70% of the historical maximum? Justify this both graphically and with appropriate mathematical calculations.
9. Identify a business related to your trend. Explain a potential commercial impact these models could have and how this may affect a commercial decision for the business.

Part 4 – evaluation

Students need to critically evaluate the models produced in parts 2 and 3 to determine an argument to the driving question, ‘Can mathematics predict periodic phenomena?’

Students need to structure a response that refers to the modelling and mathematical methods or concepts detailed in this assessment.

As part of the evaluation, students should provide

* A list of pros versus cons for each model, with references made to examples in the assessment. Any limitations of the models should be listed here.
* Suggestions for improvement, if they exist, justified by mathematics.

The final justification statement should answer the driving question by referencing the statements above.

What to submit

* Evidence of an authentic simulation. This may take the form of screenshots of the models with annotations.
* All data collected and generated from the simulation presented using appropriate tables.
* All formula, working and calculations required, either written by hand or typed. If screenshots have been provided, the formulas used need to be clearly annotated.
* All reasoning and justification, either written by hand or typed.

Success criteria

| Fluency, understanding and communication | Problem solving, reasoning and justification |
| --- | --- |

| Criteria | Working towards developing | Developing | Developed | Well developed | Highly developed |
| --- | --- | --- | --- | --- | --- |
| Part 1: Sound waves (1-2)  **MA12-5** | Students are able to develop and graph trigonometric functions. | Students are able to identify the period, frequency and amplitude of the trigonometric functions. | Students are able to develop and graph trigonometric functions which meet the specified properties. |  |  |
| Part 2: Develop the models (1-3)  **MA12-1 MA12-5 MA12-9 MA12-10** | Students are able to demonstrate the skills necessary to develop a basic model. | Students are able to develop a basic model with reference to some features of the data set. | Students are able to develop a basic model with reference to the features of the data set. | Students are able to develop two models to fit the data set with reference to the features of the data set. | Students are able to develop two models to fit the data set with reference to the features of the data set and justified using appropriate mathematical calculations. |
| Part 2: Predictions (4)  **MA12-5** |  | Students use their model to make predictions but with some conceptual flaws. | Students are able to accurately use their model to make predictions |  |  |
| Part 2: Chose a model (5)  MA12-10 |  | Students select the more accurate model. | Students select the more accurate model with pros and/or cons of each model listed. | Students select the more accurate model by comparing and contrasting pros and cons but with incomplete justification conveyed. | Students justify their choice of model by comparing and contrasting pros and cons with complete justification conveyed. |
| Part 2: Fishing times (6)  MA12-5 |  | Students interpret their model to select appropriate times but with some flaws in conceptual understanding. | Students interpret their model to select accurate times. |  |  |
| Part 3: Develop the model (1-5)  **MA12-1 MA12-5 MA12-9 MA12-10** | Students are able to identify a cyclical trend and collect data. | Students are able to develop a basic model with reference to some features of the data set. | Students are able to develop a model with their choice explained reference to the features of the data set. | Students are able to develop a model with their choices explained and supported by appropriate mathematical calculations and limitations and potential improvements identified. |  |
| Part 3: Predictions (6-8)  **MA12-9 MA12-10** |  | Students use their model to make predictions but with some conceptual flaws. | Students are able to accurately use their model to make predictions both graphically and algebraically. | Students are able to accurately use their model to make predictions both graphically and algebraically and explain any differences. |  |
| Part 3: Commercial impact (9)  **MA12-10** |  | Identify an affected business and potential impact of the trend. | Explain the impact this model could have for a company. | Explain the impact this model could have for a company and how it may affect a commercial decision. |  |
| Part 4: Evaluation  **MA12-1 MA12-5 MA12-9 MA12-10** |  |  | Students are able to provide an exhaustive list of pros and cons the models. | They are able to form a generalised statement from the pros and cons, limitations and potential for improvement | Students provide a convincing argument to support their opinion to the driving question. |

Note**s**

* Any non-attempt in a section will be deemed zero. Marks can only be attributed to attempted responses.
* Corresponding question numbers are shown in brackets.

Note to staff

The success criteria above has been designed for students and staff alike to use. Students should be presented the rubric as part of the assessment task package. Students and staff follow the process of the task downwards through the rubric and the depth of responses, for each element, across the rubric. Students should be encouraged to use the rubric to self-assess their progress as an assessment-as-learning strategy.

The aim of the assessment task is to develop students’ deep content knowledge. This is reflected in the descriptors, **working towards developing** through to **highly developed**. The level of skill and understanding required in each part of the task is different; some parts require **highly developed** or **well-developed** skills, other parts only capture a **developing** skill set.

None of the working mathematically elements are distinct and when demonstrating one element, you are invariably demonstrating another. As an example, communication runs concurrently through all the other working mathematically elements. Students cannot respond to this assessment without communicating in some form. However, it is envisaged that there is a general progression through the working mathematically elements, starting with fluency and leading to understanding, problem solving, reasoning and justification, with increasingly higher levels of communication accompanying each element. Careful consideration has been given to the position of the success criteria statements so they reflect the working mathematically elements demonstrated.

This assessment task has been designed to illuminate the style of questions and the types of responses needed to elicit deep content knowledge, however, staff are encouraged to use and adapt the assessment task and the success criteria to their school context. Staff may like to enhance or amend sections of the task. Staff may like to adapt the rubric to assign marks to the descriptors in order to differentiate between responses that address the same statement. All changes are the responsibility of the staff using the assessment.