 Effective annual rate of interest

Consider an interest rate of 6% p.a. compounded monthly.

This is 0.5% per month.

The following method can be used to calculate an effective annual rate of interest.

Consider the initial value of 1.

Option 1: Define $T\_{n}$ as the value of the investment after n months.

$$T\_{1}=1×1.005=1.005$$

$$T\_{2}=1.005×1.005$$

$$T\_{3}=1.005×1.005×1.005=1.005×1.005^{2}$$

Which is a geometric sequence with a first term, $a=1.005$, and common ratio, $r=1.005$

$$…$$

$$T\_{12}=1.005×1.005^{11}=1.06167$$

Effective annual rate of interest = 6.17% (2 dp)

6% p.a. compounded monthly is equivalent to 6.17% (2 dp) p.a. interest compounded annually.

Option 2: Define $T\_{n}$ as the value of the investment at the start of month n.

$$T\_{1}=1$$

$$T\_{2}=1×1.005$$

$$T\_{3}=1×1.005×1.005=1×1.005^{3}$$

Which is a geometric sequence with a first term, $a=1$, and common ratio, $r=1.005$

$T\_{13}$ will represent the value after one year (value at the start of the 13th month)

$$T\_{13}=1×1.005^{12}=1.06167$$

Effective annual rate of interest = 6.17% (2 dp)

6% p.a. compounded monthly is equivalent to 6.17% (2 dp) p.a. interest compounded annually.

Activity: Order the following investment options by calculating the effective annual interest rates.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Interest rate p.a.** | **Frequency of compounding** | **Interest rate per period** | **Effective annual interest rate** | **Order****(1 – largest return)** |
| 4% | Daily |  |  |  |
| 4% | Monthly |  |  |  |
| 4% | Quarterly |  |  |  |
| 4% | 6 monthly |  |  |  |

Activity: Order the following investment options by calculating the effective annual interest rates.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Interest rate p.a.** | **Frequency of compounding** | **Interest rate per period** | **Effective annual interest rate** | **Order****(1 – largest return)** |
| 5% | Daily |  |  |  |
| 5.05% | Monthly |  |  |  |
| 5.1% | Quarterly |  |  |  |
| 5.06% | 6 monthly |  |  |  |
| 5.04% | Daily |  |  |  |
| 5.06% | Quarterly |  |  |  |

Note: Students can check solutions using online calculators.