 Year 11 Mathematics Standard

| MS-S2 Relative Frequency and Probability | Unit duration |
| --- | --- |
| Statistical Analysis involves the collection, exploration, display, analysis and interpretation of data to identify and communicate key information.Knowledge of statistical analysis enables the careful interpretation of situations and raises awareness of contributing factors when presented with information by third parties, including the possible misrepresentation of information.Study of statistics is important in developing students’ understanding of the contribution that statistical thinking makes to decision-making in society and in the professional and personal lives of individuals. | 4 weeks |

| Subtopic focus | Outcomes |
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| The principal focus of this subtopic is to draw conclusions related to the chance that an event will occur.Students develop awareness of the broad range of applications of probability concepts in everyday life and their use in decision-making.Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students. | A student:* solves probability problems involving multistage events MS11-8
* uses appropriate technology to investigate, organise and interpret information in a range of contexts MS11-9
* justifies a response to a given problem using appropriate mathematical terminology and/or calculations MS11-10

**Related Life Skills outcomes**: MALS6-10, MALS6-13, MALS6-14 |

| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| The material in this topic builds on content from the Statistics and Probability Strand of the K–10 Mathematics syllabus, including the Stage 4 content for calculating the probability of simple and compound events and the Stage 5 content for estimating probabilities with relative frequency. | Summative assessment: Students could complete an investigative style task where they investigate the probabilities found in common games, or they could create their own game where the ‘house’ wins 60% of the time. |

All outcomes referred to in this unit come from [Mathematics Standard Stage 6](https://syllabus.nesa.nsw.edu.au/mathematics-standard-stage6/) Syllabus
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Glossary of terms

| Term | Description |
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| arrays | An array is an ordered collection of objects or numbers arranged in rows and columns. |
| complement | The complement of an event refers to when the event does NOT occur. For example, if A is the event of throwing a 5 on a die, then the complement of A, denoted by $\overbar{A}$ or A*c*, is throwing a number that is NOT 5 on a die |
| event | An event is a set of outcomes for a random experiment |
| outcomes | An outcome is a single possible result from an experiment. |
| population | The population in statistics is the entire dataset from which a statistical sample may be drawn |
| relative frequency | Relative frequency is a measure of the number of times that an event has occurred in a repeated experiment. If an event *E* occurs *r* times when a chance experiment has been repeated *n* times, then the relative frequency of *E* is: $\frac{r}{n}$ |
| sample space | The sample space of a chance experiment is the set of all possible outcomes for that experiment. |
| tree diagrams | A tree diagram is a diagram that can be used to determine the outcomes of a multistep random experiment. A probability tree diagram has the probability for each stage written on the branches. |

| Lesson sequence | Content | Suggested teaching strategies and resources  | Date and initial | Comments, feedback, additional resources used |
| --- | --- | --- | --- | --- |
| Place events on probability scaled and construct sample space for an experiment(2 lessons) | * review, understand and use the language associated with theoretical probability and relative frequency ◊ **Paperclip icon** Literacy icon
* construct a sample space for an experiment and use it to determine the number of outcomes (ACMEM154)
* review probability as a measure of the ‘likely chance of occurrence’ of an event (ACMMM052)
* review the probability scale: $0\leq P(A)\leq 1$ for each event $A$, with $P\left(A\right)=0$ if $A$ is an impossibility and $P\left(A\right)=1$ if $A$ is a certainty (ACMMM053)
 | Language of probability* Students need to ensure they understand how to calculate the basic probability and represent the probability in an appropriate numerical form.
* Teacher to show the [Unbelievable highway accident](https://www.youtube.com/watch?v=CEYoU5BhGl4) video as an introduction to probability.
* Teacher to create a scale on the board or using a piece of string hung across the classroom going from 0 to 1. Give students a post-it note and have them place a probability word (pre-prepared or made up by the students) in an appropriate place in the scale. The activity could be extended to include fractions/decimals/percentages that the students put on the scale too. [Probability Washing Line](http://www.transum.org/Maths/Activity/Probability/Discuss.asp)
* Teachers could compare the students’ results with those of 1700 people displayed at [If you say something is likely, how likely do people think it is?](http://www.probabilitysurvey.com/results)
* Student activity: Students identify/construct the sample space for some common experiments in probability. For example; rolling a die/two dice, cards in a normal deck, flipping coins
* Student activity: Students investigate the game [Odds and Evens](https://nrich.maths.org/4308). Is it a fair game?
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| Calculate probabilities for experiments(2 lessons) | * determine the probabilities associated with simple games and experiments ◊ **Paperclip icon**
* use the following definition of probability of an event where outcomes are equally likely: $P(event)=\frac{number of favourable outcomes}{total number of outcomes}$
 | Calculating simple probabilities* Student activity: Students play ‘Beano’ as outlined below. Have them calculate the probability of rolling each total and then determine a winning strategy.
* Each student has a game board and 12 cubes/beans/counters. They place the twelve cubes above the numbers they predict will be rolled (two dice will be rolled and the faces added).
* When the dice are rolled, if there is a cube above the sum then the student takes it away. Whoever clears the board first wins.
* Student activity: Students play Greedy Pig as outlined below. Discuss with students their strategy and the reason for their strategy. How often should a ‘1’ appear in theory? How often does is actually appear in practise?
* The aim of the game is to get the highest score after 10 rounds.
* Students start by standing up. Each time the die is rolled, you must add the number on the die to the sum of your previous rolls.
* A player can 'sit down' at any time. When a player sits, they keep all the points they have earned in the round, but are not able to earn more points until the next round.
* When a one is rolled, all 'standing' players lose the points they have accumulated in the current round.
* The player with the most points at the end of 10 rounds wins.
* Student activity: Students to investigate a situation where they are to toss a coin 20 times.
* What do they expect to happen?
* They toss 4 heads on their first 4 throws, what do they think will happen in the next 16 throws?
* Student A thinks they will still end up with 10 heads and 10 tails
* Student B thinks they will end up with 12 heads and 8 tails
* Who is correct? Student to toss a coin until they get 4 heads in a row. They then need to record what they get in the next 16 throws, to prove which student is correct.
* Student activity: Students investigate the probability of winning various [Dice Games](http://www.mathwire.com/data/dicetoss1.html) to determine whether they are ‘fair’ or not
* Student activity: Students could create their own game/experiment and record the probabilities of the different outcomes. They could be asked to make the game fair or biased and justify how they achieved this.
* The teacher could pose some reverse questions by giving a probability and have students come up with an event that matches. For example;

$P\left(event\right)=\frac{2}{13}$ (Students may say that the event is selecting an Ace or King from a deck of cards.Exemplar question:Jo and Lee each have a spinner. Jo’s spinner can land on 1, 2, 3, 4 or 5 and Lee’s spinner can land on A, B, C, or D. They spin their spinners simultaneously.* 1. Use an array to list all possible outcomes.
	2. What is the probability that the spinners show a D and an even number?

**Resource:** ms-s2-nesa-exemplar-question-solutions.DOCX |  |  |
| Calculate the probability of complementary events(1 lesson) | * + calculate the probability of the complement of an event using the relationship $P(an event does not occur)=1-P(the event does occur)=P(\overbar{the event does occur})=P(event^{c})$
 | Introducing complementary events* Student activity: Students could be familiarised with complementary events using concrete materials.
* Students have a pack of playing cards and are asked to find the complement to drawing a red card. The red cards are put in a pile and the remaining pile is the complement to drawing a red card (which is drawing a black card).
* The teacher could then complete with other pairs of complementary events (such as drawing a 7 or drawing an even number).
* Student activity: Students complete the [Complementary events treasure hunt](https://mathslinks.net/faculty/complementary-events-treasure-hunt) by starting at any poster; read the question; and then find the answer on another poster; they then read the question on that poster and so on.

**Exemplar Question**In a collection of DVDs, five are rated ‘PG’, three are rated ‘G’, and two are rated ‘M’. If a DVD is selected at random from the collection, what is the probability that it is not rated ‘M’? **Resource:** ms-s2-nesa-exemplar-question-solutions.DOCX |  |  |
| Draws probability tree diagrams(2 lessons) | * use arrays and tree diagrams to determine the outcomes and probabilities for multistage experiments (ACMEM156) **AAM** **Paperclip icon**
* construct and use tree diagrams to establish the outcomes for a simple multistage event
* use probability tree diagrams to solve problems involving two-stage events
 | Introducing tree diagrams* Students are provided with a definition and example of a multi-stage event. Students then brainstorm what types of experiments may be considered as multi-stage experiments and how many stages they have. Examples of multi stage events may be: tossing two coins, selecting one boy and one girl from a mixed class, selecting a high school student to represent the Newcastle school district by first selecting the high school, then the year group, then the student.

Using tree diagrams to calculate probability of multi-stage events* Student activity: Students create the sample space for a two-up game and then determine the chances of winning [Rules of Two Up](http://www.dailytelegraph.com.au/newslocal/inner-west/come-in-spinner-learn-how-to-play-twoup-in-time-for-anzac-day/news-story/f16f8ef4df540d166f8a0b7ea7cac834)
* Student activity: Students draw a tree-diagram to determine the sample space of a scissor-paper-rock game.
* They then determine if there is such thing as a winning strategy.
* What happens to the probabilities and to the game if we introduce a well where both the stone and scissors fall into the well and lose to it, but paper covers both stone and well.
* Another variation of the game was seen in “The Big Bang Theory” - rock paper scissor Spock lizard where Spock smashes scissors and vaporizes rock; he is poisoned by lizard and disproved by paper. Lizard poisons Spock and eats paper; it is crushed by rock and decapitated by scissors. Is this still a fair game?

**Exemplar question:**Lou and Ali are on a fitness program for one month. The probability that Lou will finish the program successfully is 0.7, while the probability that Ali will finish it successfully is 0.6. The probability tree diagram shows this information.The first stage of the tree diagram tree diagram divides into two branches. The first branch leads to Lou successful and there is a label of 0.7 on the branch. The second branch leads to Lou not successful and there is a label of 0.3 on the branch.  The second stage of the tree diagram divides into two branches from each of the previous branches. In each case, one branch indicates Ali successful and is labelled 0.6, and the other is labelled Ali not successful and is labelled 0.4What is the probability that only one of Lou and Ali will be successful?**Resource:** ms-s2-nesa-exemplar-question-solutions.DOCX |  |  |
| Uses simulations to solve problems(2 lessons) | * solve problems involving simulations or trials of experiments in a variety of contexts **AAM** ◊ **Paperclip icon**
* perform simulations of experiments using technology (ACMEM150)  Information and communication technology capability icon
* use relative frequency as an estimate of probability (ACMEM152)
* recognise that an increasing number of trials produces relative frequencies that gradually become closer in value to the theoretical probability  Information and communication technology capability icon
* identify factors that could complicate the simulation of real-world events (ACMEM153)
 | Using simulations to solve problems* Student activity: Students complete a practical experiment rolling dice 60 times. Students then investigate and describe the effect of increasing the number of trials. Results from all students can then be pooled to observe the effect of increasing the number of trials of the experiment. Students should be able to observe that experimental and theoretical probability begin to align.
* Students use a spreadsheet to simulate the flipping of a coin/rolling a dice, and so on. Students investigate the difference between the expected probability and the experimental probability and what happens as the number of trials increases. [Probability simulations involving Spreadsheets](http://www.mathmammoth.com/lessons/probability_simulations.php)
* Student activity: Students could conduct another dice rolling game “Roll a 6”. Students count how many times it takes to roll a 6 on a single die. Record this number and repeat. Consider, what might happen as you play this game over and over again. If 1000 people play this game, what would the average number of rolls needed be? What would the median be? Which would be larger/smaller?
* Student activity: Students use a spreadsheet to simulate Lotto. They use their simulation to see the most commonly and least commonly selected numbers before comparing their results with historical data.**Resource:** lotto-simulation.DOCX, lotto-simulation-example.XLXS
* Student activity: Students simulate the chances of them passing their Learner driver test by using dice or a spreadsheet**Resource:** rolling-with-the-dice.DOCX
* Student activity: Students to investigate the Monty Hall problem by using a [simulation](http://math.andyou.com/tools/montyhallsimulator/montysim.htm) to see how many times they win if the “switch” as opposed to keeping their original choice.
* Students could try and draw a tree diagram to verify their results.
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| Calculates the relative and expected frequencies of events(3 lessons) | * solve problems involving probability and/or relative frequency in a variety of contexts **AAM** **Paperclip icon**
* use existing known probabilities, or estimates based on relative frequencies to calculate expected frequency for a given sample or population, eg predicting, by calculation, the number of people of each blood type in a population given a two-way table of percentage breakdowns
* calculate the expected frequency of an event occurring using $np$ where $n$ represents the number of times an experiment is repeated, and on each of those times the probability that the event occurs is $p$
 | Relative and expected frequency* Student activity: Students investigate [Blood types by country](http://www.rhesusnegative.net/themission/bloodtypefrequencies/) and use this data to solve problems around the number of people with certain blood types in particular populations. Students can calculate the expected number of people in each blood group when given the percentage of people with that blood type.
* The [Australian Red Cross](https://www.donateblood.com.au/learn/about-blood) contains more information about what the different blood types are used for.
* Student activity: Students play roulette and determine the theoretical probability of each type of bet compared to its payoutResource: roulette.DOCX
* Student activity: Students answer the question [How risky is life?](https://www.resolve.edu.au/mathematical-modelling-how-risky-life?lesson=1672) by looking at the probability of unusual causes of death and working out their expected frequency for the population of Australia.
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Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.