 Year 12 Mathematics Extension 1

Assessment task

ME-P1 Mathematical Induction

Driving question

How does mathematical induction help us?

Outcomes

* **ME12-1** applies techniques involving proof or calculus to model and solve problems
* **ME12-6** chooses and uses appropriate technology to solve problems in a range of contexts
* **ME12 7** evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

All outcomes referred to in this unit come from [Mathematics Extension 1](https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) Syllabus © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2017

Learning across the curriculum

General capabilities

* Critical and creative thinking 
* Information and communication technology capability 
* Literacy 
* Numeracy 

Task

Select one of the following problems or one of your own choosing from your own research, of a situation where mathematical induction can be applied:

* Minimum moves in the Tower of Hanoi problem,
* The number of closed regions formed by intersecting lines,
* The number of handshakes at a conference,
* The number of diagonals in a polygon.

You are to construct a 1000 word (minimum) report that presents the following findings, including any required evidence:

1. Investigate the problem itself. Explain the problem, the solution for early values of n (a minimum of 4 values) and a description of the pattern that occurs, including a general term.
2. Construct an extensive list of the solutions for at least the first 50 values of n. Provide evidence for how this was constructed, including evidence of any software used, such as screenshots or a digital file upon submission.
3. Using a google search or otherwise, identify the rule that gives the pattern that you have found. Verify this rule by substituting a range of values for n (minimum 3) including at least one larger value from Part 2.
4. Use Mathematical Induction to prove that the series you have found is identical to the rule you have found.
5. In your own words, and with reference to the “initial statement” and the “inductive step” explain how the process of Mathematical Induction works and how it is useful in this circumstance. In what ways are we in a better position now to understand the presented problem than we were after step 2? Refer specifically to your work in this process and use appropriate mathematical terms.

Success criteria

| Fluency, understanding and communication | Problem solving, reasoning and justification |
| --- | --- |

| Criteria | Working towards developing | Developing | Developed | Well developed | Highly developed |
| --- | --- | --- | --- | --- | --- |
| 1. Investigate the problem**ME12-1** | Explains the problem and successfully calculates the solution for less than 4 values of *n* | Explains the problem clearly and successfully calculates the solution for 4 or more values of *n*. Calculates a general solution.  |  |  |  |
| 2. Extensive list of solutions**ME12-6** | Construct s a list of correct solutions for less than 50 values of *n*. | Constructs an extensive list of correct solutions for 50 or more values of *n*, using efficient methods. | Constructs an extensive list of correct solutions for 50 or more values of *n*, using the most efficient methods. Includes evidence of the use of technology to aid calculations. |  |  |
| 3. Identify the correct rule**ME12-1** | Identifies the correct rule. | Identifies the correct rule, and provides sufficient evidence of calculations.  |  |  |  |
| 4. Applies Mathematical Induction**ME12-1** | Attempts a mathematical induction proof, correctly identifying the processes for the three steps.  | Presents an incomplete Mathematical Induction proof with correctly identified steps.  | Presents a correct and complete Mathematical Induction Proof, providing adequate working and reasoning at each step.  |  |  |
| 6. Evaluates mathematical induction**ME12-7** | Attempts to explain the process of mathematical induction, matching the terms correctly to step 1 and step 2. Identifies that Mathematical Induction has been helpful in matching a rule to a series.  | Attempts to explain the process of mathematical induction, matching the terms correctly to step 1 and step 2, with some accurate explanation.Identifies that Mathematical Induction has been helpful in matching a rule to a series, and explains clearly how a rule can be more useful.  | Explains the process of Mathematical Induction clearly, using the terminology effectively to discuss how the three steps are related. Clearly conveys the role of Mathematical induction in proving the pattern and the rule are the same for all integer values.  | Explains the process of Mathematical Induction clearly, using the terminology effectively to discuss how the three steps are related. Presents a sound argument including evidence from their investigation as to the value of knowing a rule rather than the series, identifying this as Mathematical Induction’s role.  | Explains the process of Mathematical Induction extensively, referring to steps using appropriate terminology to convey the relationship between each stage, and how this results in a conclusive proof. Presents a highly convincing argument as to the value of knowing a rule rather than a series, identifying this as Mathematical Induction’s role. Uses advanced mathematical reasoning to justify their conclusion of the value of a rule.  |

Note**s**

* Any non-attempt in a section will be deemed zero. Marks can only be attributed to attempted responses.
* Corresponding question numbers are shown in brackets.

Note to staff

The success criteria above has been designed for students and staff alike to use. Students should be presented the rubric as part of the assessment task package. Students and staff follow the process of the task downwards through the rubric and the depth of responses, for each element, across the rubric. Students should be encouraged to use the rubric to self-assess their progress as an assessment-as-learning strategy.

The aim of the assessment task is to develop students’ deep content knowledge. This is reflected in the descriptors, **working towards developing** through to **highly developed**. The level of skill and understanding required in each part of the task is different; some parts require **highly developed** or **well-developed** skills, other parts only capture a **developing** skill set.

None of the working mathematically elements are distinct and when demonstrating one element, you are invariably demonstrating another. As an example, communication runs concurrently through all the other working mathematically elements. Students cannot respond to this assessment without communicating in some form. However, it is envisaged that there is a general progression through the working mathematically elements, starting with fluency and leading to understanding, problem solving, reasoning and justification, with increasingly higher levels of communication accompanying each element. Careful consideration has been given to the position of the success criteria statements so they reflect the working mathematically elements demonstrated.

This assessment task has been designed to illuminate the style of questions and the types of responses needed to elicit deep content knowledge, however, staff are encouraged to use and adapt the assessment task and the success criteria to their school context. Staff may like to enhance or amend sections of the task. Staff may like to adapt the rubric to assign marks to the descriptors in order to differentiate between responses that address the same statement. All changes are the responsibility of the staff using the assessment.