 Minimum spanning trees

A network diagram is shown below:



Trees

A **tree** is an undirected network in which any two vertices are connected by exactly one path.



Activities

1. Draw either a network with the given specification or explain why no such network exists:
2. Network, nine vertices, nine edges
3. Network, four vertices, seven edges. What is special about this network?
4. Network, five vertices, three edges
5. Tree, four vertices, three edges
6. Tree, nine vertices, nine edges
7. How many edges does a tree with “n” number of vertices have?
8. What happens if you delete an edge in a tree?
9. In a tree, can vertices be connected by more than one path?
10. Can there be ‘cycles’ in a tree?
11. What happens if you add an edge to a tree?
12. If the total degree of a network is the sum of all the degrees of its vertices, what is the total degree if a tree with ‘n’ vertices?

Spanning trees

A **spanning tree** in a network is a tree that contains each vertex. There is only one possible path between any two vertices.



The **minimum spanning tree** is the spanning tree with the minimum weight.

Minimum spanning trees

1. Find the minimum spanning tree for the network diagram below.



Could you write down a general strategy to solve the problem that works in every case? Would you need more examples to test it?

Activities

Test your strategy/strategies to find the minimum spanning tree in each case below:

















Solutions

Trees

Activities

1. Draw either a network with the given specification or explain why no such network exists:
2. Network, nine vertices, nine edges

3. Network, four vertices, seven edges. What is special about this network?


The network must have a loop or two paths joining the same two vertices. The network is not simple.

1. Network, five vertices, three edges
This is not possible, a maximum of 4 vertices can be combined into a network with 3 edges.



1. Tree, four vertices, three edges



1. Tree, nine vertices, nine edges
This is not possible. A tree with 9 vertices will have exactly 8 edges. By adding a 9th edge a cycle or loop is formed and it is therefore not a tree.
2. How many edges does a tree with “n” number of vertices have?
(n-1) edges
3. What happens if you delete an edge in a tree?
If an edge is deleted from to a tree there are two possibilities.

Option 1: If the edge deleted joined two vertices such as A and F, then it will be split two trees.
Option 2: If the edge deleted joined two vertices such as F and D, then a new tree will be formed with one less vertex.
4. In a tree, can vertices be connected by more than one path?
No.
5. Can there be ‘cycles’ in a tree?
There cannot be cycles in a tree as then there would be more than one path between two vertices.
6. What happens if you add an edge to a tree?
If an edge is added to a tree there are two possibilities.

Option 1: If the edge joins two vertices which are already part of the tree then a cycle will be formed and it will no longer represent a tree. For example, if an edge joining F and B was added to the diagram.
Option 2: If the edge joins to a vertex not previously within the tree a new tree will be formed with an additional vertex. For example, if an edge joining B and C was added to the diagram.
7. If the total degree of a graph is the sum of all the degrees of its vertices, what is the total degree if a tree with ‘n’ vertices?
$2n-2$.
Explanation: With one vertex there is no degree. When a vertex is added the total degree will increase by two.

Minimum spanning trees

1. Find the minimum spanning tree for the network diagram below.



Could you write down a general strategy to solve the problem that works in every case? Would you need more examples to test it?

**Solution:** Various responses could be given. The students responses may lead to the developments of Prim’s and Kruskal’s algorithms.

Activities

Test your strategy/strategies to find the minimum spanning tree in each case below:

**Solution:** See sample solutions for the minimum spanning trees in the resource prims-and-kruskals-algorithms.DOCX