 Arithmetic in polar and exponential form

Multiplication of complex numbers

1. Consider two complex numbers$ s=p(\cos(α)+i\sin(α))$ and $t=q(\cos(β)+i\sin(β))$.

Show that $st=pq(\cos(\left(α+β\right))+i\sin((α+β)))$

1. Choose integers or surds a, b, c and d and write two complex numbers in Cartesian form, $s=a+ib$ and $t=c+id$
	1. Multiplying in Cartesian form:
2. Calculate $st$ in Cartesian form.
3. Convert $st$ into polar form.
	1. Multiplying in polar form:
4. Convert $s$ and $t$ each into polar form.
5. Use the polar form of $s$ and $t$ to calculate $st $in polar form.
	1. Multiplying in exponential form:
6. Convert $s$ and $t$ each into exponential form
7. Use the exponential form of $s$ and $t$ to calculate $st $in exponential form.
8. Convert the exponential form of $st$ into polar form.
9. Geometric interpretation.
	1. You have two complex numbers, $s$ and $t,$ on an Argand plane.

Using only a ruler and protractor or their equivalence within graphing software, explain how you can use the result from question 1 to locate and determine the complex number $st$

* 1. Confirming your explanation:
1. Plot two complex numbers, $s$ and $t$, on a Cartesian plane.
2. Use your method from part a) to plot the complex number $st$.
3. Convert $s$ and $t$ into polar form.
4. Use the polar form of $s$ and $t$ to calculate $st $in polar form.
5. Plot $st$ from the polar form to confirm your result.

Division of complex numbers

1. Consider two complex numbers$ s=p(\cos(α)+i\sin(α))$ and $t=q(\cos(β)+i\sin(β))$.

Show that $\frac{s}{t}=\frac{p}{q}(\cos(\left(α-β\right))+i\sin((α-β)))$

1. Choose integers or surds a, b, c and d and write two complex numbers in Cartesian form, $s=a+ib$ and $t=c+id$
	1. Dividing in Cartesian form:
2. Calculate $\frac{s}{t}$ in Cartesian form.
3. Convert $\frac{s}{t}$ into polar form.
	1. Dividing in polar form:
4. Convert $s$ and $t$ each into polar form.
5. Use the polar form of $s$ and $t$ to calculate $\frac{s}{t} $in polar form.
	1. Dividing in exponential form:
6. Convert $s$ and $t$ each into exponential form
7. Use the exponential form of $s$ and $t$ to calculate $\frac{s}{t} $in exponential form.
8. Convert the exponential form of $\frac{s}{t}$ into polar form.
9. Geometric interpretation.
	1. You have two complex numbers, $s$ and $t$, on an Argand plane.

Using only a ruler and protractor or their equivalence within graphing software, explain how you can use the result from question 1 to locate and determine the complex number $\frac{s}{t}$.

* 1. Confirming your explanation:
1. Plot two complex numbers, $s$ and $t$, on a Cartesian plane.
2. Use your method from part a) to plot the complex number $\frac{s}{t}$.
3. Convert $s$ and $t$ into polar form.
4. Use the polar form of $s$ and $t$ to calculate $\frac{s}{t} $in polar form.
5. Plot $st$ from the polar form to confirm your result.

Powers of complex numbers

1. Consider the complex number$ z=a+ib=r\left(\cos(θ)+i\sin(θ)\right)=re^{iθ}$
	1. Using Cartesian form, calculate :
2. $z^{2}$
3. $z^{3}$
4. $z^{-1}$
5. $z^{-2}$
6. $z^{-3}$
	1. Using polar form, calculate :
7. $z^{2}$
8. $z^{3}$
9. $z^{-1}$
10. $z^{-2}$
11. $z^{-3}$
	1. Using exponential form, calculate :
12. $z^{2}$
13. $z^{3}$
14. $z^{-1}$
15. $z^{-2}$
16. $z^{-3}$
17. Choose integers or surds a and b, given the complex number $z=a+ib$, express the complex number in Cartesian, polar and exponential form.
	1. Using Cartesian form, calculate :
18. $z^{2}$
19. $z^{3}$
20. $z^{-1}$
21. $z^{-2}$
22. $z^{-3}$
	1. Using polar form, calculate:
23. $z^{2}$
24. $z^{3}$
25. $z^{-1}$
26. $z^{-2}$
27. $z^{-3}$
	1. Using exponential form, calculate :
28. $z^{2}$
29. $z^{3}$
30. $z^{-1}$
31. $z^{-2}$
32. $z^{-3}$
33. Geometric interpretation.
	1. You have a complex number, $s$, on an Argand plane.
	2. Using only a ruler and protractor or their equivalence within graphing software, explain how you can use the result from question 1b to locate and determine the complex number $s^{n}$ where $n$ is an integer.
	3. Confirming your explanation:
34. Plot a complex number, $s$, on a Cartesian plane.
35. Use your method from part a) to plot the complex number $s^{4}$.
36. Convert $s$ into polar form.
37. Use the polar form of $s$ to calculate $s^{4} $in polar form.
38. Plot $s^{4}$ from the polar form to confirm your result.